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Subresultants, Sylvester sums and the rational interpolation problem ☆

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ABSTRACT

We present a solution for the classical univariate rational interpolation problem by means of (univariate) subresultants. In the case of Cauchy interpolation (interpolation without multiplicities), we give explicit formulas for the solution in terms of symmetric functions of the input data, generalizing the well-known formulas for Lagrange interpolation. In the case of the osculatory rational interpolation (interpolation with multiplicities), we give determinantal expressions in terms of the input data, making explicit some matrix formulations that can independently be derived from previous results by Beckermann and Labahn.

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1. Introduction

The *Cauchy interpolation problem* or rational interpolation problem, considered already in Cauchy (1841), Rosenhain (1845), Predonzan (1953), is the following:

Let K be a field, $a, b \in \mathbb{Z}_{\geq 0}$, and set $\ell = a + b$. Given a set $\{x_0, \ldots, x_\ell\}$ of $\ell + 1$ distinct points in K, and $y_0, \ldots, y_\ell \in K$, determine—if possible—polynomials $A, B \in K[x]$ such that $\deg(A) \leq a$, $\deg(B) \leq b$

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and

$$\frac{A}{B}(x_i) = y_i, \quad 0 \le i \le \ell. \tag{1}$$

This might be considered as a generalization of the classical Lagrange interpolation problem for polynomials, where $b = 0$ and $a = \ell$. In contrast with that case, there is not always a solution to this problem, since for instance by setting $y_0 = \cdots = y_a = 0$, the numerator A is forced to be identically zero, and therefore the remaining y_{a+k}, $1 \le k \le \ell - a$, have to be zero as well. However, when there is a solution, then the rational function A/B is unique as shown below.

The obvious generalization of the Cauchy interpolation problem receives the name *osculatory rational interpolation problem* or rational Hermite interpolation problem:

Let K be a field, $a, b \in \mathbb{Z}_{\ge 0}$, and set $\ell = a + b$. Given a set $\{x_0, \ldots, x_k\}$ of $k + 1$ distinct points in K, $a_0, \ldots, a_k \in \mathbb{Z}_{\ge 0}$ such that $a_0 + \cdots + a_k = \ell + 1$, and $y_{i,j} \in K$, $0 \le i \le k$, $0 \le j < a_i$, determine—if possible—polynomials $A, B \in K[x]$ such that $\deg(A) \le a$, $\deg(B) \le b$ and

$$\left(\frac{A}{B}\right)^{(j)}(x_i) = j!\,y_{i,j}, \quad 0 \le i \le k,\ 0 \le j < a_i. \tag{2}$$

This problem has also been extensively studied from both an algorithmic and theoretical point of view, see for instance Salzer (1962), Kahng (1969), Wuytack (1975), Beckermann and Labahn (2000), Tan and Fang (2000) and the references therein. A unified framework, which relates the rational interpolation problem with the Euclidean algorithm, is presented in Antoulas (1988), and also in the book von zur Gathen and Gerhard (2003, Section 5.7), where it is called *rational function reconstruction*. In Theorem 2.2 below, we translate these results to the subresultants context, which enables us to obtain some explicit expressions in terms of the input data for both problems.

For the Cauchy interpolation problem, there exists an explicit closed formula in terms of the input data that can be derived from the results on symmetric operators in a suitable ring of polynomials presented in Lascoux (2003), as shown in Lascoux (2013). Theorem 3.1 recovers this expression from the relationship between subresultants and the Sylvester sums introduced in Sylvester (1853), see also Lascoux and Pragacz (2003), D'Andrea et al. (2007, 2009), Roy and Szpirglas (2011), Krick and Szanto (2012).

We also present in Theorem 4.2 an explicit determinantal expression for the solution of the osculatory rational interpolation problem in terms of the input data, giving it as a quotient of determinants of generalized Vandermonde-type (and Wronskian-type) matrices. This generalizes straight-forwardly the corresponding known determinantal expression for the classical Hermite interpolation problem, setting another unified framework for all these interpolation problems. As mentioned in Remark 4.4 below, this determinantal expression can actually also be derived following the work of Beckermann and Labahn (2000), as we concluded from a recent useful discussion with George Labahn.

Since no closed formula for subresultants in terms of roots with multiplicities is known yet—except for very few exceptions, see D'Andrea et al. (2013)—a generalization of Theorem 3.1 to the osculatory rational interpolation problem is still missing, and some more work on the subject must be done in order to shed light to the problem.

2. Subresultants and the rational interpolation problem

Let us start by showing that a solution A/B for the rational interpolation problem, when it exists, is unique.

Proposition 2.1. *If the osculatory rational interpolation problem (2) has a solution, then there exists a unique pair (A, B) with $\gcd(A, B) = 1$ and A monic such that A/B is a solution.*

Proof. If there is a solution, then, cleaning common factors and dividing by the leading coefficient of A, there is a solution satisfying the same degree bounds with $\gcd(A, B) = 1$ and A monic. Assume

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