Journal of Symbolic Computation 68 (2015) 146–168

Contents lists available at ScienceDirect

Journal of Symbolic Computation

www.elsevier.com/locate/jsc

Quantifier elimination for a class of exponential polynomial formulas ☆

Ming Xu [a], Zhi-Bin Li [a], Lu Yang [b,c]

[a] Department of Computer Science and Technology, East China Normal University, Shanghai 200241, China
[b] Chengdu Institute of Computer Applications, Chinese Academy of Sciences, Chengdu 610041, China
[c] Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai 200062, China

A R T I C L E I N F O

Article history:
Received 27 October 2013
Accepted 22 July 2014
Available online 20 September 2014

MSC:
03C10
14Q15
68W30

Keywords:
Exponential polynomials
Cylindrical algebraic decomposition
Quantifier elimination
Decision procedures
Interval arithmetic

A B S T R A C T

Quantifier elimination is a foundational issue in the field of algebraic and logic computation. In first-order logic, every formula is well composed of atomic formulas by negation, conjunction, disjunction, and introducing quantifiers. It is often made quite complicated by the occurrences of quantifiers and nonlinear functions in atomic formulas. In this paper, we study a class of quantified exponential polynomial formulas extending polynomial ones, which allows the exponential to appear in the first variable. We then design a quantifier elimination procedure for these formulas. It adopts the scheme of cylindrical decomposition that consists of four phases—projection, isolation, lifting, and solution formula construction. For the non-algebraic representation, the triangular systems are introduced to define transcendental coordinates of sample points. Based on that, our cylindrical decomposition produces projections for input variables only. Hence the procedure is direct and effective.

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☆ This research was supported in part by NSFC (No. 11401218, No. 11071273, & No. 61021004), NBRPC (No. 2011CB302402), SRFDP (No. 20130076120010), and the Open Project of Shanghai Key Laboratory of Trustworthy Computing (No. 07dz22304201307).

E-mail addresses: mxu@cs.ecnu.edu.cn (M. Xu), lizb@cs.ecnu.edu.cn (Z.-B. Li), luyang@casit.ac.cn (L. Yang).

http://dx.doi.org/10.1016/j.jsc.2014.09.015
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1. Introduction

Quantifier elimination is a foundational issue in the field of algebraic and logic computation with extremely wide applications, such as computer-aided geometric design, program verification and testing, control synthesis of dynamic and hybrid systems, to name just a few. As is well known, in first-order logic, every formula is well composed of atomic formulas by a finite times of negation, conjunction, disjunction, and introducing quantifiers. It is often made quite complicated by the occurrences of quantifiers and nonlinear functions (particularly transcendental functions) in atomic formulas. Once all quantified variables are eliminated, the resulting formula would be simply understood for both human and machine. Such elimination, however, is not easy in general due to the complex structures of varieties of nonlinear functions. Hence **quantifier elimination** on nonlinear formulas is a significant and challenging job.

In 1930s, Tarski invented a quantifier elimination method for the elementary theory of real closed fields $\mathfrak{T}(\mathbb{R}; <, =; +, \cdot; 0, 1)$ (Tarski, 1951). Seidenberg (1954) and Cohen (1969) offered two alternative methods. However, these methods required too much computation to be practical except for quite trivial instances. In 1973, Collins presented the first practical quantifier elimination method—**cylindrical algebraic decomposition (CAD)** for $\mathfrak{T}(\mathbb{R}; <, =; +, \cdot; 0, 1)$ (Collins, 1975). It splits the whole space into a finite number of connected regions, on each of which the input formula is truth-invariant, and yields the complete solution formula by collecting the defining formulas of feasible regions. (The details will be recalled in Subsection 2.2.) Its complexity is double-exponential w.r.t. the number of input variables. So Collins and his descendants were devoted to improving the efficiency of CADs. An important breakthrough was the **partial CAD** (Collins and Hong, 1991) that utilized three parts of information in the input formula (the quantifiers, the Boolean structure, and the absence of some variables in some atomic formulas) to partially build the decomposition. Besides, an approximate quantifier elimination method was studied in Hong and Safey El Din (2009, 2012). It required the input formula to satisfy a certain extra condition, and allowed the solution formula to be almost equivalent to the input formula. Thus it could successfully tackle some challenging problems, such as stability analysis of the renowned MacCormack's scheme. For the other school, Weispfenning (1988) presented another practical quantifier elimination method for linear polynomial formulas by virtual substitution of linear expressions, and pointed out that the complexity of such linear problems has a double-exponential lower bound w.r.t. the number of input variables too. Thus the theoretical complexity of the quantifier elimination problem for $\mathfrak{T}(\mathbb{R}; <, =; +, \cdot; 0, 1)$ was established. Later Weispfenning (1997) developed this method for the quadratic case by virtual substitution of square-root expressions and the general case by Thom's lemma (Coste and Roy, 1988). The above mature methods were implemented in the computer algebra tools QEPCAD (Brown, 2003) and Redlog (Dolzmann and Sturm, 1997), respectively.

Besides, Tarski also concerned whether the decidability result could be extended to the theory $\mathfrak{T}(\mathbb{R}; <, =; +, \cdot, \exp; 0, 1)$, i.e. introducing the exponential function. Unfortunately, the extended theory was proven to admit no quantifier elimination (van den Dries, 1982), and was shown to be decidable when Schanuel's conjecture holds (Wilkie, 1996). The known positive results thereby focused on some sub-theories of $\mathfrak{T}(\mathbb{R}; <, =; +, \cdot, \exp; 0, 1)$. Richardson (1991) investigated univariate exponential polynomials, and devised the so-called false/pseudo-derivative sequences for estimating the numbers of real roots of them. The result was an overestimate because some non-real roots were not ruled out then. Maignan (1998) applied this method to a class of bivariate exponential polynomial equations. Recently, Achatz et al. (2008) first presented a complete algorithm to isolating distinct real roots of univariate exponential polynomials. Their main idea is to isolate real roots of the original function when real roots of the simpler pseudo-derivative have been isolated. The termination is guaranteed by Lindemann's theorem. McCallum and Weispfenning (2012) varied this isolation algorithm for two similar functions obtained by replacing the exponential function with the logarithm function and the inverse tangent function, respectively, and proposed the decision procedure for multivariate sentences transcendental only in the first variable (i.e. the theories $\mathfrak{T}_{\exp}$, $\mathfrak{T}_{\ln}$, $\mathfrak{T}_{\arctan}$ to be specified later). Meanwhile, Strzeboński (2008, 2009) studied the real root isolation of larger classes of univariate transcendental functions—exp–log functions and tame elementary functions, respectively. But the termination of the isolation algorithm depends on Schanuel's conjecture (Richardson, 1997).

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