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A generic position based method for real root isolation of zero-dimensional polynomial systems

Jin-San Cheng, Kai Jin

KLMM, Institute of Systems Science, AMSS, Chinese Academy of Sciences, China

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ABSTRACT

We improve the local generic position method for isolating the real roots of a zero-dimensional bivariate polynomial system with two polynomials and extend the method to general zero-dimensional polynomial systems. The method mainly involves resultant computation and real root isolation of univariate polynomial equations. The roots of the system have a linear univariate representation. The complexity of the method is $\tilde{O}_B(N^{10})$ for the bivariate case, where $N = \max(d, \tau)$, d resp., τ is an upper bound on the degree, resp., the maximal coefficient bitsize of the input polynomials. The algorithm is certified with probability 1 in the multivariate case. The implementation shows that the method is efficient, especially for bivariate polynomial systems.

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1. Introduction

Real root isolation of zero-dimensional polynomial systems is a fundamental problem in symbolic computation and it has many applications. The problem has been studied for a long time and there are a lot of results. One can compute the real roots of a zero-dimensional polynomial system by symbolic methods, numeric methods and symbolic–numeric methods. In context of symbolic methods, we can mention the characteristic set methods, Gröbner basis methods, the resultant methods and so on. In this paper, we focus on the resultant methods. We consider the zero-dimensional system as $\{f_1, \ldots, f_m\} \subset \mathbb{Z}[x_1, \ldots, x_n]$, where $\mathbb{Z}$ is the ring of integers.

E-mail addresses: jcheng@amss.ac.cn (J.-S. Cheng), jinkaijl@163.com (K. Jin).

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The idea of this paper comes from a geometric property of the roots of a polynomial system: generic position. Generic position was used in the polynomial system solving for a long time (Alonso et al., 1996; Becker and Wörmann, 1996; Canny, 1988; Cheng et al., 2009; Diochnos et al., 2009; Giusti et al., 2001; Gao and Chou, 1999; Giusti and Heintz, 1991; Kobayashi et al., 1988; Rouillier, 1999; Tan and Zhang, 2009; Yokoyama et al., 1989). Let's explain it for the bivariate case. Simply speaking, a zero-dimensional bivariate system is said to be in a **generic position** if we can find a complex plane, say the x-axis, such that different complex zeros of the system are projected to different complex points on the complex x-axis. In the rest of this paper, when we say root(s), we mean real root(s) if there is no special illustration.

Solving bivariate polynomial systems is widely studied in recent years (Busé et al., 2005; Cheng et al., 2009; Corless et al., 1997; Emiris et al., 2008; Emiris and Tsigaridas, 2005; Diochnos et al., 2009; Emeliyanenko et al., 2011; Hong et al., 2008; Qin et al., 2013). Most of these methods projected the systems to two directions (x-axis, y-axis) and identified whether a root pair (one x-coordinate and one y-coordinate) was a true root or not (Diochnos et al., 2009; Emeliyanenko et al., 2011; Hong et al., 2008; Qin et al., 2013). In Busé et al. (2005), Corless et al. (1997), they projected the roots of the bivariate system to x-axis, using a matrix formulation, and lifted them up to recover the roots of the original system. The multiplicity of the roots are also considered.

A **local generic position** method was proposed to isolate the real roots of a zero-dimensional bivariate polynomial system in Cheng et al. (2009). In the local generic position method, the roots of a zero-dimensional bivariate polynomial system $\Sigma = \{f(x,y), g(x,y)\}$ are represented as linear combinations of the roots of two univariate polynomial equations $R_1(x) = \mathrm{Res}_y(f,g) = 0$ and $R_2(x) = \mathrm{Res}_y(f(x+sy,y), g(x+sy,y)) = 0$:

$$\left\{x=\alpha,\ y=\frac{\beta-\alpha}{-s}\ \middle|\ \alpha\in\mathbb{V}\big(R_1(x)\big),\ \beta\in\mathbb{V}\big(R_2(x)\big),\ |\beta-\alpha|<S\right\},$$

where s, S are constants satisfying certain given conditions. Each root (α,β) of $\Sigma = 0$ is projected in $R_2(x) = 0$ such that the corresponding root is in a neighborhood of $\alpha : E = \{v \mid |v-\alpha| < S\}$. All the roots of $R_2(x) = 0$ in E correspond to the roots of $\Sigma = 0$ on the fiber $x = \alpha$. Thus we can recover the y-coordinates of the roots of $\Sigma = 0$ from the roots of $R_2(x) = 0$. The multiplicities of the roots of $\Sigma = 0$ are also preserved in the corresponding roots of $R_2(x) = 0$. The implementation of the method shows that it is efficient and stable when compared to the best methods at that time, especially when the system has multiple roots. But the local generic position method has a bottleneck. When some of the roots of $R_1(x)$ are very close, s will be very small. Thus computing $R_2(x)$ and isolating its roots is time-consuming. Sometimes, it is more than 90% of the total computing time! The rate increases when the degrees of the polynomials in the systems increase.

The contribution of the paper is that we present a method to overcome the bottleneck of the local generic position method and extend the method to general zero-dimensional multivariate polynomial system mainly involving resultant computation and univariate polynomial root isolation, which is easy to implement. We also analyze the complexity of the algorithm for the bivariate case. We compare our implementation with several other efficient related softwares, such as local generic position method (Cheng et al., 2009), Hybird method (Hong et al., 2008), Discovery (Xia and Yang, 2002) and Isolate (Rouillier, 1999). The results show that our algorithm is efficient, especially in bivariate case.

In order to overcome the drawback of the local generic position method, we present a method to search for a better s with a small bitsize and present another way to recover the roots of the system. This is the main contribution of the paper. Finding the correspondence between the roots of $\Sigma = 0$ and $R_2(x) = 0$, we can recover the roots of $\Sigma = 0$. It works as follows. First, we compute $R_1(x)$ and its roots. From the isolating intervals of the roots of $R_1(x) = 0$, we get the root isolating interval candidates of $f = g = 0$ by computing the roots of interval polynomials. We compute a rational number s such that any two isolating interval candidates are not overlapping under a linear transformation $\varphi : (x,y) \to (x-sy, y)$ and $\{\varphi(f), \varphi(g)\}$ is in a generic position. Then for each isolating interval candidate $K = [a,b] \times [c,d]$, we can isolate the roots of $R_2(x) = 0$ in the interval $\pi_y(\varphi(K))$ ($\pi_y : (x,y) \to (x)$) to recover the isolating intervals of $f = g = 0$. The multiplicity(ies) of the root(s) of the system in K is (are) the multiplicity(ies) of the corresponding root(s) in $\pi_y(\varphi(K))$. The bivariate

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