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A dynamic stochastic decision-making method based on discrete time sequences

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ABSTRACT

This paper studies a dynamic stochastic decision-making method based on discrete time sequences. With the aim of rectifying the deficiency of calculation methods that use only the "sequence" information in the determination of discrete time sequence weight, we first propose a discrete time sequence weight calculation method that can deal with unequal time intervals by introducing an exponential decay model, and gather the original dynamic stochastic decision information according to the timing weight and attribute weight. Second, considering the disadvantages of decision criteria such as traditional stochastic dominance, stochastic multi-criteria acceptability analysis, and connection number of set pair analysis, in the absence of original information and limited conditions, we propose a decision method based on possibility, convert this information into interval numbers based on the theory of stochastic probability distribution, and then rank the order of the scheme using the possibility model of interval numbers. Finally, we test the effectiveness and reasonableness of the method by empirical calculation and show that the proposed method is both practical and effective.

1. Introduction

Multi-attribute decision making is a selection problem that considers multiple attributes. Because of the complexity and uncertainty of multi-attribute decision-making problems, the measuring of attribute values or the evaluation results may be in the form of stochastic variables; one of the most common forms is that the attribute values obey or approximately obey the normal distribution. For example, product life, product acceptability, and the demands of customers or markets sometimes obey the normal distribution [1–3]. How to solve the problem of multi-attribute decision making with normal stochastic variables is of vital significance.

Stochastic decision making with multiple stochastic variable attribute values meeting a certain probability distribution is an important branch in the field of uncertain multi-criteria decision making. Of wide concern to scholars, the problem of stochastic decision making has mainly been dealt with using approaches based on stochastic dominance [1–3], SMAA (stochastic multi-criteria acceptability analysis) [4–6], the connection number of set pair analysis, prospect theory [7–10], prospect stochastic dominance [11], probability weighted means (PWM) [12], etc. These kinds of methods provide broader ideas for the field of stochastic multi-attribute decision making and expand research boundaries, but there are some inevitable defects in these methods, due to the limitation of objective conditions. For example, the method of stochastic dominance to resolve stochastic multi-attribute decision-making problems with normal random variables can only qualitatively determine two dominant relationship parts between the two schemes and cannot determine the degree of the dominance; based on the Monte Carlo Simulation, SMAA obtains the ranking results in the sense of a certain degree of confidence; set pair analysis studies things and their system just through three aspects: similarity, difference, and contrariety; prospect stochastic dominance is difficult to use to accurately quantify a person's psychological behaviour and is greatly influenced by the environment; PWM only considers the estimated probability information and is too general and rough compared with a random dominant decision. So it loses a lot of original information in the decision-making process, influencing the accuracy.

In addition, the similar methods above are all based on a single static time sequence environment, and in the actual decision problems, where the attribute values are normalized stochastic variables like stock value forecasts, evaluation of sports competitive rivals, and customer demand preferences, it is necessary to take account of decision-making information from the past or over several periods of the present, so that the decision made will be more comprehensive and reasonable. Traditional approaches only consider adding a time dimension in the stochastic decision-making

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problem of attribute dimension and objective dimension (this is called the dynamic stochastic decision-making problem). How to determine the time sequence weight has become the key to solving the dynamic decision-making problem. At present, there is less research literature on dynamic stochastic decision making; however, the results from research on dynamic multi-attribute decision making based on fuzzy theory have been quite rich, and the research is focused on how to obtain the time sequence weight, mainly using the arithmetic progression and geometric progression methods [13], normal distribution method [14], binomial distribution method [15], index distribution method [16], principle of maximum entropy [17], and time sequence ideal solution method [18]. These methods provide a theoretical basis for further research on multi-attribute decision-making problems and enrich the dynamic decision-making theory, but only a few scholars have made progress in analysing the dynamic random decision-making problem based on the above time sequence weight determination methods [7,19–21]. However, such methods only extract the continuity information of the time sequence, and ignore the influence of discrete time sequences like specific time change and unequal time intervals on the time sequence weight. In the random dynamic decision-making problem where the attribute value can be approximately described with normal distribution, the information mastered by the decision makers varies greatly in different moments: as time passes, approaching the final decision-making moment, the richer the information accumulation is, the larger the influence on the final decision-making result, and vice versa. Therefore, if the time sequence is assigned a weight based only on the consecutive ordinal number for a sequence with unequal time intervals, the weight will not be objective, and this will influence the scientific decision-making result.

Based on this, a new method for solving the dynamic stochastic decision-making problem is proposed under the discrete time sequence environment. To address the insufficiency of decision criteria in the literature [1–10], this method puts forward a possibility distribution matrix based on the theory of stochastic probability distribution, which allows the decision-making process to avoid a large amount of information loss, making the decision results more precise and more widely applicable. To address the deficiency of sequential weight calculation in the literature [13–18], we introduce time sequence parameters and, according to the theory of population ecology, present a weight calculation method of discrete time sequence based on a damped exponential model; the method considers the influence on time sequence weight from discrete time series such as time intervals, and can reflect the process of dynamic stochastic decision-making more objectively and comprehensively.

The structure is as follows: Section 2 draws the related basic concepts and algorithms of criteria; Section 3 constructs the dynamic stochastic decision-making model based on the context of discrete time sequence and the theory of possibility; Section 4 verifies the rationality and effectiveness of this method with an example.

2. Preparation

Let the probability density of a continuous stochastic variable be $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, where $x \in [-\infty, +\infty]$, parameters μ and σ^2 respectively represent the expectancy and variance of the stochastic variable, and X is called a normal distribution, recorded as $X \sim N(\mu,\ \sigma^2)$, with accumulated probability function $F(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{x} e^{-(t-\mu)^2/2\sigma^2} dt$. When $\mu = 0$ and $\sigma = 1$, X submits to the normal distribution of standardization, and the corresponding probability density and accumulated distribution function are, respectively, $\vartheta(x) = \frac{1}{\sqrt{2\pi}} e^{x^2/2}$ and $\theta(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \vartheta(t) dt$. For convenience, let $\{\mu,\ \sigma\}$ denote the normal distribution number of stochastic variable X, recorded as $x = \{\mu, \sigma\}$, and let Θ denote the set of all normal distribution numbers.

Definition 1. [1]: If X is a stochastic variable, then according to the 3σ principle of probability statistics of stochastic events, the probability of variable X at $[\mu - 3\sigma, \mu + 3\sigma]$ is:

$$P\{\mu - 3\sigma < X < \mu + 3\sigma\} = 99.74\% \tag{1}$$

Definition 2. [3]: For any two normal distribution numbers $x_1 = \{\mu_1, \sigma_1\}$ and $x_2 = \{\mu_2, \sigma_2\}$:

(1) $x_1 \oplus x_2 = \{\mu_1 + \mu_2, \sigma_1 + \sigma_2\}$;
(2) $\lambda x_1 = \{\lambda\mu_1, \lambda\sigma_1\}$;
(3) $\lambda(x_1 \oplus x_2) = \lambda x_1 \oplus \lambda x_2$;

Definition 3. [19]: For a set of normal distribution numbers given by $x_j = \{\mu_j, \sigma_j\}$, and NDNWAA: $\Theta^n \to \Theta$,

$$NDNWAA_w(x_1, x_2, \ldots, x_n) = w_1x_1 \oplus w_2x_2 \oplus \cdots \oplus w_nx_n = \left\{ \sum_{j=1}^{n} w_j\mu_j, \sqrt{\sum_{j=1}^{n} w_j^2\sigma_j^2} \right\} \tag{2}$$

is called the weighted arithmetic mean operator of the normal distribution number, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weighted vector of the attribute of random variable $x_j (j = 1, 2, \ldots, n)$, $w_j \in [0,\ 1]$, $\sum w_j = 1$.

Definition 4. [19]: Let t be the time variable such that at the moment t, the random variable X submits to the normal distribution $N(\mu(t),\ (\sigma(t))^2)$; let its normal distribution number be $\{\mu(t),\ \sigma(t)\}$, recorded as $x(t) = \{\mu(t), \sigma(t)\}$.

Definition 5. [19]: Let $x(t) = (x(t_1), x(t_2), \ldots, x(t_p))$ be the normal distribution number in p different moments $t_k (k = 1, 2, \ldots, p)$. Then

$$\begin{aligned} &DNDNWAA_{w(t)}(x(t_1), x(t_2), \ldots, x(t_p)) \\ &= w(t_1)x(t_1) \oplus w(t_2)x(t_2) \oplus \cdots \oplus w(t_p)x(t_p) \\ &= \left\{ \sum_{k=1}^{p} w(t_k)\mu(t_k), \sqrt{\sum_{k=1}^{p} (w(t_k))^2 (\mu(t_k))^2} \right\} \end{aligned} \tag{3}$$

is called the weighted arithmetic mean operator of the normal distribution number, where $w(t) = (w(t_1), w(t_2), \ldots, w(t_p))^T$ is the weight of time sequence $t_k (k = 1, 2, \ldots, p)$, $w(t_k) \in [0,\ 1]$, and $\sum_{k=1}^{p} w(t_k) = 1$.

3. Dynamic random decision-making model based on discrete time sequences

To address the problem of dynamic stochastic multi-attribute decision making, it is very important to determine the time weight and to make the decision criteria reasonable, in order to rank the alternative schemes effectively and scientifically. Most of the studies currently determine the time weight in the hypothesis condition of continuous time sequence and then rank the alternative schemes, but there are certain deficiencies in the existing stochastic decision-making criteria. Therefore, based on discrete time sequence, in a departure from previous research, this paper puts forward a dynamic stochastic decision-making model that applies the time-damped exponential to determine the time sequence weight, and highlights the effects on the decision results from different interval time sequence weights; meanwhile, the possibility matrix determines the stochastic decision rules. Further, we optimize the process of dynamic multi-attribute decision making and make the decision results more reasonable. The construction model is as follows.