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The bi-objective quadratic multiple knapsack problem: Model and heuristics

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ABSTRACT

The single objective quadratic multiple knapsack problem (QMKP) is a useful model to formulate a number of practical problems. However, it is not suitable for situations where more than one objective needs to be considered. In this paper, we extend the single objective QMKP to the bi-objective case such that we simultaneously maximize the total profit of the items packed into the knapsacks and the 'makespan' (the gain of the least profit knapsack). Given the imposing computational challenge, we propose a hybrid two-stage (HTS) algorithm to approximate the Pareto front of the bi-objective QMKP. HTS combines two different and complementary search methods — scalarizing memetic search (first stage) and Pareto local search (second stage). Experimental assessments on a set of 60 problem instances show that HTS dominates a standard multi-objective evolutionary algorithm (NSGA II), and two simplified variants of HTS. We also present a comparison with two state-of-the-art algorithms for the single objective QMKP to assess the quality of the extreme solutions of the approximated Pareto front.

1. Introduction

Given a set of weight capacity-constrained knapsacks and a set of objects (or items), each object is associated with a weight, an individual profit and a paired profit with any other object. The quadratic multiple knapsack problem (QMKP) aims to determine a maximum profit assignment (packing plan) of objects to the knapsacks subject to their capacity constraints [14]. The profit of a pair is accumulated in the sum only if the two corresponding objects are allocated to the same knapsack.

The QMKP generalizes two well-known knapsack problems, i.e., the quadratic knapsack problem (QKP) [28] and the multiple knapsack problem (MKP) [26]. The QMKP is also related to the so-called discounted 0–1 knapsack problem (DKP) [30]. The DKP is to select items from a set of groups where each group includes three items and at most one of the three items can be selected. For each group, the profit of the third item is a discounted profit which is defined by the interaction of the first two items. This problem remains linear and finds applications in investment. The QMKP has important applications where resources with different levels of interaction have to be distributed among different tasks [14,31]. A first example involves allocating staff in a company to a set of groups where group member contributions are calculated both individually and in pairs; another example concerns an investment scenario where the knapsacks represent the bounded budgets and the paired values indicate the impact on expected financial performances when different investment options are chosen together.

The QMKP is computational challenging since it generalizes the NP-hard QKP [28]. To solve such problems, exact and heuristic algorithms constitute two main and complementary solution methods in the literature. Exact algorithms have the theoretical advantage of guaranteeing the optimality of the solutions found. However given the intrinsic difficulty of NP-hard problems, the computing time needed to find the optimal solution by an exact algorithm may become prohibitive for large instances. For instance, for the QKP, the most powerful exact algorithm can only deal with instances with no more than 1500 items [29] and typically requires considerable computing time. For the more complicated QMKP, no exact algorithm has been published in the literature to the best of our knowledge. On the other hand, heuristic algorithms aim to find satisfactory sub-optimal solutions (to large problem instances) in acceptable computing time, but without provable quality guarantee of the attained solutions. This approach is particularly useful when it is difficult or impossible to obtain an optimal solution. Within the context of approximating the difficult QMKP, several heuristic algorithms have been reported in the literature. Representative heuristic algorithms include population-based algorithms, such as genetic algorithms [14,31], memetic algorithm

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[32], artificial bee colony algorithm [33] and evolutionary path relinking algorithm [8]. Besides, neighborhood search and constructive/destructive search approaches represent another class of effective tools for the QMKP; typical examples include hill-climbing [14], tabu-enhanced iterated greedy search [13], strategic oscillation [12] and iterated responsive threshold search (IRTS) [7]. Finally, note that exact and heuristic approaches complement each other and are useful for problem solving in different settings. They can even be combined to create powerful hybrid algorithms.

The QMKP is a useful model to formulate a number of practical problems [14,31]. Still it is not suitable for situations where more than one objective needs to be simultaneously considered. Consider the following staff assignment problem for instance. When company managers allocate staff to form a set of groups responsible for different products, they may not only consider the total strength of all groups, but also the balance among the groups for the sake of fairness and sustainable development. For instance, when several groups work on different products destined to different types of customers, in order to pursue a long-term profit, company managers may want to balance the strength of each group when allocating the group members, such that each group has enough capability to ensure a high-quality of its products for the purpose of well satisfying its customers in the long term. In portfolio investments, an investor may be interested not only by maximizing the total return of the invested asset mix, but also by ensuring an expected return of the least profit asset. In these settings, one faces a bi-objective version of the QMKP, in which both the total profit of the packing plan and the gain of the least profit knapsack (corresponding to the makespan in scheduling theory) are to be maximized simultaneously (see Section 2 for the formal definition). To conveniently formulate this type of problems, this work extends the single objective QMKP to the bi-objective QMKP (BO-QMKP). Apart from the aforementioned application scenarios, the BO-QMKP model could find in the future additional applications in other settings where it can be used to formulate either a whole problem or a subproblem. One notices that some well-known knapsack problems and more generally other optimization problems have already a bi-objective or multi-objective counterpart, like the bi-objective 0–1 knapsack problem [10], the multi-objective multidimensional knapsack problem [4], the bi-objective unconstrained binary quadratic programming problem [21], the bi-objective flow-shop scheduling problems [17], the bi-objective traveling salesman problem [20], the multi-objective set covering problem [15] and the bi-objective capacity planning problem [36]. The BO-QMKP introduced in this work enriches these multi-objective modeling tools and enlarges the class of practical problems that can be formulated.

On the other hand, solving the BO-QMKP model represents an imposing computational challenge in the general case since it generalizes the computationally difficult QMKP model. For this reason, we focuses on elaborating a heuristic algorithm to approximate the Paretofront of the BO-QMKP. The proposed hybrid two-stage (HTS) algorithm is based on the general two-stage approach combining two fundamentally different and complementary search strategies, namely the scalarizing approach (first stage) and the Pareto-based approach (second stage). Such a hybrid framework has been successfully applied to solve a number of challenging multi-objective problems such as the bi-objective flow-shop scheduling problems [17], the multi-objective traveling salesman problem [20] and the bi-objective unconstrained binary quadratic programming problem [22]. In this work, we adapt this general two-stage approach to solve our BO-QMKP model and develop dedicated search procedures for each stage of the proposed algorithm. In particular, we devise a population-based scalarizing memetic search method to effectively solve the scalarizing subproblems in the first stage and a double-neighborhood Pareto local search procedure to further the approximation set in the second stage. By combining complementary search strategies in the two search stages and using dedicated techniques in each stage, the HTS algorithm aims to push the approximation set towards the Pareto front on the one hand and ensure a well-distributed approximation set on the other hand. Thanks to these desirable features, the proposed HTS algorithm proves to be able to attain high quality approximations as shown in Section 4.

The main contributions of this work can be summarized as follows.

- We introduce for the first time the bi-objective quadratic multiple knapsack model whose formal definition is provided in Section 2.
- We provide a detailed description of our hybrid two-stage approach which aims to provide high quality approximation of the Pareto front for the proposed BO-QMKP model (Section 3). We show how HTS makes an original combination of an elitist evolutionary multi-objective optimization algorithm with a state-of-the-art single-objective responsive threshold search procedure for its first stage while adopting an effective Pareto-based local search procedure for the second stage.
- We show experimental studies on a set of 60 benchmark instances to assess the effectiveness of the proposed HTS algorithm (Section 4). In particular, our experiments demonstrate that HTS dominates a conventional non-dominated sorting genetic algorithm (NSGA II) and two simplified variants of the HTS algorithm.

2. The bi-objective quadratic multiple knapsack problem

In this section, we introduce formally the BO-QMKP model. Given a set of objects (or items) $N = \{1, 2, ..., n\}$ and a set of capacity-constrained knapsacks $M = \{1, 2, ..., m\}$. Each object i ($i \in N$) is associated with a profit p_i and a weight w_i. Each pair of objects i and j ($1 \le i \neq j \le n$) is associated with a joint profit p_{ij}. Each knapsack k ($k \in M$) has a weight capacity C_k. The BO-QMKP aims to assign the n objects to the m knapsacks (some objects can remain unassigned) such that both the overall profit of the assigned objects and the makespan (the gain of the least profit knapsack) are maximized subject to the following two constraints:

- Each object i ($i \in N$) can be allocated to at most one knapsack;
- The total weight of the objects assigned to each knapsack k ($k \in M$) cannot exceed its capacity C_k.

Given the above notations, a BO-QMKP solution can be represented as a set of groups $S = \{I_0, I_1, \ldots, I_m\}$ where group $I_k \subset N$ ($k \in M$) represents the set of objects assigned to knapsack k and group I_0 contains all unassigned objects. Then the BO-QMKP can be stated mathematically as follows:

$$max \quad f_1(S) = \sum_{k \in M} \sum_{i \in I_k} p_i + \sum_{k \in M} \sum_{i,j \in I_k, i \neq j} p_{ij} \tag{1}$$

$$max \quad f_2(S) = min_{k \in M} \left\{ \sum_{i \in I_k} p_i + \sum_{i,j \in I_k, i \neq j} p_{ij} \right\} \tag{2}$$

subject to:

$$\sum_{i \in I_k} w_i \le C_k, \forall k \in M \tag{3}$$

$$S \in \{0, ..., m\}^n \tag{4}$$

Eq. (1) aims to maximize the total profit of all assigned objects while Eq. (2) aims to maximize the gain of the least profit knapsack (or makespan). Constraints (3) guarantees that the total