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# Uncertain mean-variance and mean-semivariance models for optimal project selection and scheduling

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ABSTRACT

This paper discusses a joint problem of optimal project selection and scheduling in the situation where initial outlays and net cash inflows of projects are given by experts' estimates due to lack of historical data. Uncertain variables are used to describe these parameters and the use of them is justified. A new mean-variance and a mean-semivariance models are proposed considering relationship and time sequence order between projects. In order to solve the complex problems, the methods for calculating uncertain lower partial semivariance and higher partial semivariance values are introduced and a hybrid intelligent algorithm which integrates genetic algorithm with cellular automation is provided. In addition, two examples are presented to illustrate the application and significance of the new models, and numerical experiments are done to show the effectiveness of the proposed algorithm.



## 1. Introduction

The original project selection refers to selecting an appropriate combination of projects among available ones to obtain the maximal total profit within budget limitation. A main contribution to the problem was made by Weingartner [38] who first introduced mathematical programming method into the field. Since then, a variety of models were developed to increase applicability of the proposed models to the real life, as discussed by Dickinson et al. [5], Gutjahr et al. [11], Liu and Wang [26], Padberg and Wilczak [31], Xiao et al. [39], etc.

These studies treated the project parameters as exact values, yet it is usually difficult to get the exact numbers of them because of the complexity of real world. Therefore, scholars studied the project selection problem with imprecise project parameter values. Traditionally, people employed probability theory to handle the problems. For example, De et al. [4], Keown and Martin [18], Keown and Taylor [19] initiated chance-constrained programming methods to deal with the project selection problem with random inflows and outlays. Medaglia et al. [29] put forward a new evolutionary method for solving linearly constrained project selection problems. Shakhsi-Niaei et al. [34] employed Monte Carlo simulation to propose a two phases framework for project selection problem under randomness and subject to real-world constraints.

Although probability theory is powerful for handling indeterminacy, its use is only suitable when people have sufficient historical data such that the probability distributions can be obtained. However, there are situations in real life where there are scarce or no historical data. This is especially true for the selection of R & D projects whose initial costs of research and the incomes brought about by the new products have no observed data and can only be estimated by the experts. A great deal of evidence shows that people usually include a wider range of values in their estimation of an indeterminant number than it may really take. Then in that situation if we still employ probability theory to help make decisions, counterintuitive results may appear. The examples can be found in Liu [24] and Zhang et al. [41]. Especially, Zhang et al. [41] shows that in this situation if we inappropriately used probability theory in project selection, a budget exceeding event which is sure to happen would be judged the event that will surely not happen. Considering that people will not take action for an event that will never happen, this mistaken result may bring great loss to the investors.

To deal with men's estimates, scholars have studied employing fuzzy set theory and have applied fuzzy set theory in different fields [27,42]. To solve project selection problems, scholars have developed a variety of fuzzy models, eg., Bhattacharyya et al. [1], Huang [12], Karsak and Kuzgunkaya [17], Tsao [36], Zhang et al. [43] etc. These researches opened a new perspective for dealing with project selection problems with parameters given by experts' estimations. However, recently it was found that paradoxes will occur if we use fuzzy variables to describe the subjective estimations of project parameters [40,41]. In order to model human being's imprecise estimations toward indeterminant quantities, Liu [25] founded an uncertainty theory based on four axioms and refined it in Liu [23]. If we use uncertainty theory to model human being's imprecise

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estimations, no paradoxes appear. In fact, Liu [25] has shown that human beings' estimations expressed by belief degrees satisfy the four axioms of the uncertainty theory, which implies that we can use uncertainty theory to model human uncertainty. So far, uncertainty theory has been used to handle many optimization problems concerning human uncertainty. For example, Liu [21] proposed a spectrum of uncertain programming models and applied them to solve machine scheduling, vehicle routing and project scheduling problems [23]. In 2010, Huang [13] first employed uncertainty theory to propose a theory of uncertain portfolio selection. Based on the risk measurement of variance, Zhang et al. [45] discussed two uncertain portfolio selection models. Applications of uncertainty theory can also be found in many other optimization fields, e.g., shortest path problem [7], facility location problem [8], Chinese postman problem [44], single-period inventory problem [32], uncertain aggregate production planning problem [30], and multi-product newsboy problem [6], etc. In the area of project selection, Zhang et al. [40] first applied uncertainty theory to solve a multinational project selection problem. Zhang et al. [41] later proposed a profit risk index and a cost overrun risk index and developed an uncertain mean-risk index domestic project selection model. In this paper we will explore using uncertainty theory to solve a joint problem of optimal project selection and scheduling with initial outlays and net cash inflows given by experts' estimates. Different from previous uncertain project selection studies [40,41], in our problem, not only the projects need to be selected, but also the start times of the selected projects need to be scheduled to ensure effective use of budget. The logical relations such as independent, dependent, exclusive, and time sequence order among the candidate projects will be considered. The objective is to get the maximum profit under the capital exceeding risk and profit risk control. With an essential innovation, a new mean-variance and mean-semivariance models for project selection and scheduling in different situations will be developed. To solve the proposed complex mixed integer programming problem, a hybrid intelligent algorithm will be presented. The experimental test results will show that the algorithm can improve the convergence speed and effectively solve the problem.

The paper proceeds as follows. In Section 2 we will review some fundamentals of uncertainty theory which will be used in the paper. In Section 3 we will develop a mean-variance and a mean-semivariance models for a project selection and scheduling problem taking different interactions among candidate projects into account. In Section 4 we will provide a hybrid intelligent algorithm for solving the proposed problem. To illustrate the modeling idea and to show the effectiveness of the proposed algorithm, we will present two numerical examples and experiments in Section 5. Finally, in Section 6 we will give some concluding remarks.

## 2. Fundamentals of uncertainty theory

Uncertainty theory is developed based on the below four axioms.

**Definition 1.** Let $\mathcal{L}$ be a $\sigma$-algebra over a nonempty set $\Gamma$. Every element $\Lambda \in \mathcal{L}$ is called an event. If a set function $\mathcal{M}\{\Lambda\}$ satisfies the following four axioms, we call it an uncertain measure [22,25]:

(i) (Normality) $\mathcal{M}\{\Gamma\} = 1$.
(ii) (Duality) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$.
(iii) (Subadditivity) For every countable sequence of events $\{\Lambda_k\}$, we have

$$\mathcal{M}\left\{\bigcup_{k=1}^{\infty}\Lambda_k\right\} \le \sum_{k=1}^{\infty}\mathcal{M}\{\Lambda_k\}.$$

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space.

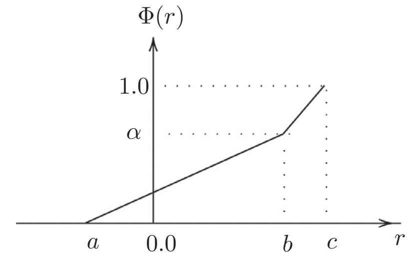


**Fig. 1.** A zigzag uncertain variable $\xi = \mathcal{Z}(a, b, c, \alpha)$.

(iv) (Product Axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots$, The product uncertain measure is

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\},$$

where $\Lambda_k$ are arbitrarily chosen events from $\mathcal{L}_k$ for $k = 1, 2, \ldots$, respectively.

**Definition 2** [25]**.** An uncertain variable is a measurable function $\xi$ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers.

It has been proved that for any events $\Lambda_1 \subset \Lambda_2$, we have

$$\mathcal{M}\{\Lambda_1\} \le \mathcal{M}\{\Lambda_2\}.$$

In application, an uncertain variable is characterized by an uncertainty distribution function which is defined as follows:

**Definition 3** [25]**.** The uncertainty distribution $\Phi$: $\Re \to [0, 1]$ of an uncertain variable $\xi$ is defined by

$$\Phi(r) = \mathcal{M}\{\xi \le r\}.$$

For example, if an uncertain variable has the following normal uncertainty distribution, we call it a normal uncertain variable:

$$\Phi(r) = \left(1 + \exp\left(\frac{\pi(\mu - r)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad r \in \Re,$$

where $\mu$ and $\sigma$ are real numbers and $\sigma > 0$. We denoted it in the paper by $\xi \sim \mathcal{N}(\mu, \sigma)$.

We call an uncertain variable a zigzag uncertain variable if it has the following zigzag uncertain distribution (Please also see Fig. 1):

$$\Phi(r) = \begin{cases} 0, & \text{if } r \le a, \\ \alpha(r-a)/(b-a), & \text{if } a < r < b, \\ (r - b - \alpha r + \alpha c)/(c-b), & \text{if } b \le r < c, \\ 1, & \text{if } r \ge c, \end{cases}$$

where $a$, $b$, $c$ and $\alpha$ are real numbers and $a < b < c$ and $\alpha \in [0, 1]$. The variable is denoted in the paper by $\xi \sim \mathcal{Z}(a, b, c, \alpha)$.

When the uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are represented by uncertainty distributions, the operational law is given by Liu [23] as follows:

**Theorem 1** [23]**.** *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$. Let $f(r_1, r_2, \ldots, r_n)$ be strictly increasing with respect to $r_1, r_2, \ldots, r_n$. Then*

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$$

*is an uncertain variable with inverse uncertainty distribution function*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)), \quad 0 < \alpha < 1. \tag{1}$$

**Theorem 2** [23]**.** *Let $\xi_1, \xi_2, \ldots, \xi_n, \xi_{n+1}, \ldots, \xi_{n+m}$ be independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, $\Phi_{n+1}, \ldots, \Phi_{n+m}$. Let $g(r_1, r_2, \ldots, r_n, r_{n+1}, \ldots, r_{n+m})$ be strictly increasing with respect to $r_1, r_2, \ldots, r_n$ and strictly decreasing with respect to $r_{n+1}, r_{n+2}, \ldots, r_{n+m}$. Then*

$$\eta = g(\xi_1, \xi_2, \ldots, \xi_n, \xi_{n+1}, \ldots, \xi_{n+m})$$