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Multiple criteria decision-making methods with completely unknown weights in hesitant fuzzy linguistic term setting

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ABSTRACT

As for multi-criteria decision making problems with hesitant fuzzy linguistic information, it is common that the criteria involved in the problems are associated with the predetermined weights, whereas the information about criteria weights is generally incomplete. This is because of the complexity and the inherent subjective nature of human thinking. In this circumstance, the weights of criteria can be derived by means of information entropy from the evaluation values of criteria for alternatives. To the best of our knowledge, up to now, there is no work having introduced the concept of entropy measure for hesitant fuzzy linguistic term sets (HFLTSs). Hence, in this paper, we are going to fill in this gap by developing information about how entropy measures of HFLTSs can be designed.

1. Introduction

Three most important research topics in the fuzzy set theory are entropy, similarity, and distance measures which have drawn the attention of many researchers who studied these concepts in practical applications, such as decision-making [6,11–14], pattern recognition [19,27], etc. The notion of entropy for fuzzy sets and their extensions allows us to measure the degree of fuzziness, ambiguity, or the uncertainty of a set which returns the amount of difficulty in making a decision whether an element belongs to that set or not. Entropy measure has received more and more attention since its appearance. Zadeh [42] proposed several entropy formulas based on Shannon's function and furthermore they put forward an axiomatic definition of entropy measure of fuzzy sets [9]. On the basis of distance between degrees of membership function of a fuzzy set and that of its nearest crisp set, Kaufmann [15] suggested an entropy measure formula for fuzzy sets. Yager [40] defined the entropy measure of a fuzzy set in terms of a lack of distinction between that fuzzy set and its complement. Over the last decades, many researchers have developed and studied entropy measures for extensions of fuzzy sets. Burillo and Bustince [3] proposed an entropy measure on interval-valued fuzzy sets and intuitionstic fuzzy sets. Different from Burillo and Bustince's [3] view point, Zeng and Li [43] presented the concept of entropy for interval-valued fuzzy sets. A nonprobabilistic entropy measure suggested by Szmidt and Kacprzyk [32] for intuitionstic fuzzy sets. In the use of the rough set theory for quantifying ambiguities in images, Sen and Pal [31] introduced classes of entropy measures for rough sets. Farhadinia [12] presented a theoretical development on the entropy of interval-valued fuzzy sets based on the intuitionistic distance and its relationship with similarity measure. Farhadinia [9] investigated the relationship between the entropy, the similarity measure and the distance measure for hesitant fuzzy sets and interval-valued hesitant fuzzy sets.

In view of the relationship between the entropy and the similarity measure for fuzzy sets and their extensions, Zeng and Guo [44] showed that a number of similarity measures and entropies for interval-valued fuzzy sets can be deduced by normalized distances of interval-valued fuzzy sets on the basis of their axiomatic definitions. Several researchers showed that similarity measures and entropies for interval-valued fuzzy sets can be transformed by each other. Zeng and Li [43] discussed the relationship between the similarity and the entropy measures of interval-valued fuzzy sets, and gave some theorems to show that the similarity and the entropy measures of interval-valued fuzzy sets can be transformed by each other based on their axiomatic definitions. Farhadinia [9] studied the systematic transformation of the entropy into the similarity measure for hesitant fuzzy sets and vice versa. For more study of the entropy and the similarity measure, the interested reader is referred to [18,36].

There exist many situations in real life where information may be unquantifiable due to its nature, for instance, in evaluating the "speed" of a car, terms like *slow, average* and *fast* are usually preferred over precise and exact values for assessing the qualitative aspect of "speed". This is because the precise quantitative information may be unavailable or the cost for its computation is too high, and thus

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linguistic terms are more close to the human cognitive processes. This shows that the use of linguistic terms makes experts judgment more reliable and informative for decision making. The linguistic approach is an approximate technique which can be used in different fields, such as, marketing [39], clinical diagnosis [7], decision making [8], risk in software development [17], education [16], technology transfer strategy selection [4], information retrieval [2], etc.

However, the implementation of linguistic approaches has some serious limitation due to the fact that these approaches assess a linguistic variable by using a single or simple linguistic terms. This kind of representation of the value of a linguistic variable may not reflect really what the decision makers mean. In many situations involving high degree of uncertainty, the decision makers might hesitant among several linguistic terms or need a complex linguistic term to represent their opinions. For example, in evaluation the "speed" of a car, one decision maker may say the speed is *not too fast*, and another decision maker may say the speed is *between average and fast*. In such cases, the traditional linguistic approaches are not able to represent such comprehensive linguistic expressions. Recently, motivated by hesitant fuzzy sets [33] and linguistic term sets, Rodriguez et al. [30] developed the hesitant fuzzy linguistic term sets (HFLTSs) to improve the modeling and computational abilities of the traditional linguistic approaches. Since the HFLTS provides a more powerful form to represent decision makers' qualitative judgments, it has attracted more and more scholars' attention. Rodriguez et al. [30] investigated some basic HFLTS operations and discussed their properties. Liao et al. [22] developed different types of distance and similarity measures for HFLTSs, and then applied them to multi-criteria decision making problems under hesitant fuzzy linguistic circumstances. Wei et al. [35] discussed the aggregation theory for HFLTS. On the basis of the pessimistic and the optimistic attitudes of the decision makers, Chen and Hong [5] presented a new method for multi-criteria linguistic decision making. For more noteworthy contributions on HFLTS applications, one can refer to [23,24,28,29,48].

However, up to now, as far as we know, there has been no report concerning the entropy measure for HFLTSs. The main objective here is to develop a theoretical framework that will assist researchers in designing entropy measures of HFLTSs. This development is based on the relationship between the entropy measures and the similarity measure for HFLTSs. Furthermore, we give a theorem that allows us to create a variety of entropies by the use of given entropies of HFLTSs.

The structure of this contribution is as follows: Section 2 reviews the concept of linguistic term sets and then presents the concept of HFLTSs. Section 3 is devoted to the results on the transformation of the distance and the similarity measures to the entropy measures for HFLTSs. Moreover, the latter section describes the procedure of entropy creation by the use of given entropies of HFLTSs. Section 4 gives the application of the proposed entropy measures to multi-criteria decision making with completely unknown weights in the HFLTS Setting. This paper is concluded in Section 5.

2. Hesitant fuzzy linguistic term sets (HFLTSs)

In decision making problems with linguistic information, experts usually feel more comfortable to express their opinions by linguistic variables (or linguistic terms) because this approach is more realistic and it is close to the human cognitive processes. In this regard, the values of variables are qualitative rather than quantitative, that is, the variable values are words or sentences instead of numbers. For example, in evaluating the "speed" of a car, linguistic labels like *very fast, fast* and *slow* are usually used because it may be unavailable for us to provide a quantitative evaluation of "speed" or the cost of evaluation of "speed" may be computationally too high. In this situation, an "approximate value" is obviously more comfortable. In order that decision makers provide their preferences over an object with linguistic labels, it needs to be predefined a proper linguistic evaluation scale. To do so, Xu [38] proposed the following finite and totally ordered discrete linguistic term set as:

$$\mathfrak{S} = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\},$$

where τ is a positive integer, and s_α represents a possible value for a linguistic variable. For example, a set of seven ($\tau = 3$) terms $\mathfrak{S}$ could be given as the following:

$$\mathfrak{S} = \{s_{-3} = \textit{very slow}, \quad s_{-2} = \textit{slow}, s_{-1} = \textit{slightly slow}, \\ s_0 = \textit{average}, \quad s_1 = \textit{slightly fast}, \quad s_2 = \textit{fast}, \quad s_3 = \textit{very fast}\}. \tag{1}$$

The mid linguistic label s_0 represents an assessment of *indifference*, and the remaining linguistic labels are symmetrically located around s_0. It is necessary that the totally ordered linguistic term set $\mathfrak{S}$ satisfies the following characteristics:

1. $s_\alpha < s_\beta$ if and only if $\alpha < \beta$;
2. The negation operator is defined as: $N(s_\alpha) = s_{-\alpha}$.

Generally, in the aggregation procedure of linguistic labels in the totally ordered linguistic term set $\mathfrak{S}$, the decision maker may deal with the aggregated result which is not match any of the original linguistic labels. In this case and to preserve all the original and the resulted linguistic labels, the discrete term set $\mathfrak{S}$ is extended to the continuous term set $\overline{\mathfrak{S}} = \{s_\alpha | \alpha \in [-q, q]\}$ where $q(q > \tau)$ is a sufficiently large positive integer. Xu [38] called $s_\alpha \in \mathfrak{S}$ the *original linguistic term*, and $s_\alpha \in \overline{\mathfrak{S}}$ the *extended (or virtual) linguistic term*. Note that the extended linguistic terms also meet the latter characteristics 1 and 2.

Based on the extended linguistic evaluation scale $\overline{\mathfrak{S}}$, the following operational laws are introduced: (see e.g. [37])

For any two linguistic terms $s_\alpha, s_\beta \in \overline{\mathfrak{S}}$, the following conditions hold

$$s_\alpha \oplus s_\beta = s_{\alpha+\beta}; \tag{2}$$

$$s_\alpha \oplus s_\beta = s_\beta \oplus s_\alpha; \tag{3}$$

$$\lambda s_\alpha = s_{\lambda\alpha}; \tag{4}$$

$$(\lambda_1 + \lambda_2) s_\alpha = \lambda_1 s_\alpha \oplus \lambda_2 s_\alpha; \tag{5}$$

$$\lambda(s_\alpha \oplus s_\beta) = \lambda s_\alpha \oplus \lambda s_\beta, \tag{6}$$

where $0 \leq \lambda, \lambda_1, \lambda_2 \leq 1$.

By the inspiration of the idea of hesitant fuzzy sets (HFSs) [33], Rodriguez et al. [30] introduced the hesitant fuzzy linguistic term set (HFLTS) to overcome some difficulties observed in a qualitative circumstance where a decision maker may hesitate between several terms at the same time, or he/she needs a complex linguistic term instead of a single linguistic term to assess a linguistic variable. Continuing that work, Liao et al. [21] refined the concept of HFLTS mathematically as follows:

Definition 2.1. Let $X = \{x_1, x_2, \ldots, x_N\}$ be a reference set, and $\mathfrak{S} = \{s_\alpha | \alpha = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a linguistic term set. A hesitant fuzzy linguistic term set (HFLTS) on X is mathematically shown in terms of

$$H_{\mathfrak{S}} = \{\langle x_i, h_{\mathfrak{S}}(x_i)\rangle | x_i \in X\}. \tag{7}$$

Here, $h_{\mathfrak{S}}(x_i)$ is a set of some possible values in the linguistic term set $\mathfrak{S}$ and can be characterized by

$$h_{\mathfrak{S}}(x_i) = \{s_{\delta_l}(x_i) | s_{\delta_l}(x_i) \in \mathfrak{S}, l = 1, 2, \ldots, L\}, \tag{8}$$

where L denotes the number of linguistic terms in $h_{\mathfrak{S}}(x_i)$.

Example 2.2. Suppose that an expert is invited to evaluate the approximate speed of three cars x_1, x_2 and x_3. Note that this criterion is