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Constructive methods of rough approximation operators and multigranulation rough sets

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ABSTRACT

Four kinds of constructive methods of rough approximation operators from existing rough sets are established, and the important conclusion is obtained: some rough sets are essentially direct applications of these constructive methods. Moreover, the new notions of non-dual multigranulation rough sets and hybrid multigranulation rough sets are introduced, and some properties are investigated.

1. Introduction

The Pawlak's rough set model [17] is based on equivalence relations, it has been generalized to arbitrary binary relations based rough sets, tolerance or similarity relations based rough sets, fuzzy rough sets and intuitionistic fuzzy rough sets (see [4,5,44]), etc. Moreover, as one of generalized models, covering rough sets has attracted much attention and induced lots of interesting results [13,19,31,34,38,40,46].

For the above rough set models, we usually only consider a single approximation space. In some directions of research on multiple-source approximation systems (see [6]), multi-agent systems or groups of intelligent agents (see [23–25,29]), multigranulation rough sets (see [7,9–12,20–22,27,36,39]), dynamic spaces and collections of general approximation spaces (see [15,16]), we need to consider multiple approximation spaces. Therefore, the algebraic structures and the relationship between various rough approximation pairs based on different approximation spaces have become an important research topic. In fact, algebra approach is widely applied in the research of rough set theory (see [2,3,8,14,18,26,28,35,37,41–43,45]). In this paper we will investigate basic algebraic operations (union and intersection) of rough approximations pairs based on multiple approximation spaces.

From another point of view, many rough set models (in particular, various multigranulation rough set models) are introduced, for these rough approximation operators, whether there are some inherent regularity or general generation rules? In this paper, we give a novel answer of the question.

The remainder of this paper is organized as follows. The next section deals with some preliminary concepts and properties regarding the Pawlak's rough sets and multigranulation rough sets. In Section 3, we introduce four kinds of constructive methods of rough approximation operations from existing rough sets, and discuss their basic properties and applications. In Section 4, we apply the constructive methods to multigranulation rough sets, and firstly introduce the new notions of non-dual multigranulation rough sets and hybrid multigranulation rough sets. We also discuss multigranulation rough sets based on general binary relations.

2. Basic concepts and properties

2.1. Pawlak's rough sets

Let U denote a non-empty set called the universe. Let $R\subseteq U\times U$ be an equivalence relation on U. The pair $apr=(U,R)$ is called an approximation space. The equivalence relation R partitions the set U into disjoint subsets. Let U/R denote the quotient set consisting of equivalence classes of R, and $[x]_R$ the equivalence class containing x. Given an arbitrary set $A\subseteq U$, in general it may not be possible to describe X precisely in (U, R). One may characterize X by a pair of lower

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and upper approximations:

$$\underline{R}(X) = \{x \in U \mid [x]_R \subseteq X\},$$

$$\overline{R}(X) = \{x \in U \mid [x]_R \cap A \neq \emptyset\}.$$

The pair $(\underline{R}(X), \overline{R}(X))$ is referred to as rough set approximation of X.

Let $\sim X = U - X$, we have the following basic properties of Pawlak's rough sets.

(L1) $\underline{R}(X) \subseteq X$	(H1) $X \subseteq \overline{R}(X)$
(L2) $\underline{R}(\emptyset) = \emptyset$	(H2) $\overline{R}(\emptyset) = \emptyset$
(L3) $\underline{R}(U) = U$	(H3) $\overline{R}(U) = U$
(L4) $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$	(H4) $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
(L5) $X \subseteq Y \Rightarrow \underline{R}(X) \subseteq \underline{R}(Y)$	(H5) $X \subseteq Y \Rightarrow \overline{R}(X) \subseteq \overline{R}(Y)$
(L6) $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$	(H6) $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$
(L7) $\underline{R}(\sim X) = \sim \overline{R}(X)$	(H7) $\overline{R}(\sim X) = \sim \underline{R}(X)$
(L8) $\underline{R}(\underline{R}(X)) = \underline{R}(X)$	(H8) $\overline{R}(\overline{R}(X)) = \overline{R}(X)$
(L9) $\underline{R}(\overline{R}(X)) = \overline{R}(X)$	(H9) $\overline{R}(\underline{R}(X)) = \underline{R}(X)$

2.2. Multigranulation rough sets

In recent years, Qian et al. [20–22] have proposed a new rough set model called multigranulation rough set. In this model, a target concept is approximated by multiple binary relations. Next, we briefly outline two definitions of multigranulation rough set models, i.e., optimistic and pessimistic multigranulation rough sets respectively. Detailed descriptions could be found in [20–22].

Definition 2.1. Let $K = (U, \mathbf{R})$ be a knowledge base, where $\mathbf{R}$ is a family of equivalence relations on the universe U. Let $A_1, A_2, \ldots, A_m \in \mathbf{R}$, where m is a natural number. For any $X \subseteq U$, its optimistic lower and upper approximations with respect to $A_1, A_2 \ldots, A_m$ are respectively defined as follows.

$$\underline{\sum_{i=1}^{m} A_i}^{O}(X) = \{x \in U \mid [x]_{A_1} \subseteq X \text{ or } [x]_{A_2} \subseteq X \text{ or } \cdots \text{ or } [x]_{A_m} \subseteq X\};$$

$$\overline{\sum_{i=1}^{m} A_i}^{O}(X) = \sim \underline{\sum_{i=1}^{m} A_i}^{O}(\sim X).$$

$(\underline{\sum_{i=1}^{m} A_i}^{O}(X), \overline{\sum_{i=1}^{m} A_i}^{O}(X))$ is called the optimistic multigranulation rough sets of X. Here, the word "optimistic" means that only one granular structure is needed to satisfy with the inclusion condition between an equivalence class and a target concept when multiple independent granular structures are available in problem processing.

Definition 2.2. Let $K = (U, \mathbf{R})$ be a knowledge base, where $\mathbf{R}$ is a family of equivalence relations on the universe U. Let $A_1, A_2, \ldots, A_m \in \mathbf{R}$, where m is a natural number. For any $X \subseteq U$, its pessimistic lower and upper approximations with respect to $A_1, A_2, \ldots, A_m$ are respectively defined as follows.

$$\underline{\sum_{i=1}^{m} A_i}^{P}(X) = \{x \in U \mid [x]_{A_1} \subseteq X \text{ and } [x]_{A_2} \subseteq X \text{ and } \cdots \text{ and } [x]_{A_m} \subseteq X\};$$

$$\overline{\sum_{i=1}^{m} A_i}^{P}(X) = \sim \underline{\sum_{i=1}^{m} A_i}^{P}(\sim X).$$

$(\underline{\sum_{i=1}^{m} A_i}^{P}(X), \overline{\sum_{i=1}^{m} A_i}^{P}(X))$ is called the pessimistic multigranulation rough sets of X. Here, the word "pessimistic" means that all granular structures are needed to satisfy with the inclusion condition between an equivalence class and a target concept when multiple independent granular structures are available.

3. Constructive methods of rough approximation operators from existing approximation operators

In this section, we establish the constructive methods of rough approximation operators from existing rough approximation operators. At first, we give some notations and preliminary results.

For convenience and unity, we use the following symbols (L1)–(L9) and (H1)–(H9) to denote the basic properties of any operator pair $(\underline{apr}, \overline{apr})$, where $\underline{apr}$ and $\overline{apr}$ are mappings from $P(U)$ to $P(U)$:

(L1) $\underline{apr}(X) \subseteq X$;
(H1) $X \subseteq \overline{apr}(X)$
(L2) $\underline{apr}(\emptyset) = \emptyset$
(H2) $\overline{apr}(\emptyset) = \emptyset$
(L3) $\underline{apr}(U) = U$
(H3) $\overline{apr}(U) = U$
(L4) $\underline{apr}(X \cap Y) = \underline{apr}(X) \cap \underline{apr}(Y)$
(L4′) $\underline{apr}(X \cap Y) \subseteq \underline{apr}(X) \cap \underline{apr}(Y)$
(H4) $\overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y)$
(H4′) $\overline{apr}(X \cup Y) \supseteq \overline{apr}(X) \cup \overline{apr}(Y)$
(L5) $X \subseteq Y \Rightarrow \underline{apr}(X) \subseteq \underline{apr}(Y)$
(H5) $X \subseteq Y \Rightarrow \overline{apr}(X) \subseteq \overline{apr}(Y)$
(L6) $\underline{apr}(X \cup Y) \supseteq \underline{apr}(X) \cup \underline{apr}(Y)$
(H6) $\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y)$
(L7) $\underline{apr}(\sim X) = \sim \overline{apr}(X)$
(H7) $\overline{apr}(\sim X) = \sim \underline{apr}(X)$
(L8) $\underline{apr}(\underline{apr}(X)) = \underline{apr}(X)$
(L8′) $\underline{apr}(\underline{apr}(X)) \supseteq \underline{apr}(X)$
(H8) $\overline{apr}(\overline{apr}(X)) = \overline{apr}(X)$
(H8′) $\overline{apr}(\overline{apr}(X)) \subseteq \overline{apr}(X)$
(L9) $\underline{apr}(\overline{apr}(X)) = \overline{apr}(X)$
(L9′) $\underline{apr}(\overline{apr}(X)) \subseteq \overline{apr}(X)$
(H9) $\overline{apr}(\underline{apr}(X)) = \underline{apr}(X)$
(H9′) $\overline{apr}(\underline{apr}(X)) \supseteq \underline{apr}(X)$

It is easy to prove the following lemma and the proof is omitted.

Lemma 3.1. *Let U be a non-empty set, $\underline{apr}$ and $\overline{apr}$ be mappings from $P(U)$ to $P(U)$. Then*

(1) *If $\underline{apr}$ satisfies (L1) for any $X \in P(U)$, then $\underline{apr}$ satisfies (L2).*
(2) *If $\overline{apr}$ satisfies (H1) for any $X \in P(U)$, then $\overline{apr}$ satisfies (H3).*
(3) *For any $X, Y \in P(U)$, if $\underline{apr}$ satisfies (L4), then $\underline{apr}$ satisfies (L5).*
(4) *For any $X, Y \in P(U)$, if $\overline{apr}$ satisfies (H4), then $\overline{apr}$ satisfies (H5).*
(5) *If $\underline{apr}$ satisfies (L1) and (L5) for any $X, Y \in P(U)$, then $\underline{apr}$ satisfies (L4′).*
(6) *If $\overline{apr}$ satisfies (H1) and (H5) for any $X, Y \in P(U)$, then $\overline{apr}$ satisfies (H4′).*
(7) *For any $X, Y \in P(U)$, if $\underline{apr}$ satisfies (L6), then $\underline{apr}$ satisfies (L5).*
(8) *For any $X, Y \in P(U)$, if $\overline{apr}$ satisfies (H6), then $\overline{apr}$ satisfies (H5).*
(9) *If $\underline{apr}$ satisfies (L5) for any $X, Y \in P(U)$, then $\underline{apr}$ satisfies (L6) for any $X, Y \in P(U)$.*
(10) *If $\overline{apr}$ satisfies (H5) for any $X, Y \in P(U)$, then $\overline{apr}$ satisfies (H6) for any $X, Y \in P(U)$.*
(11) *If $\underline{apr}$ and $\overline{apr}$ satisfy (L7) for any $X \in P(U)$, then $\underline{apr}$ and $\overline{apr}$ satisfy (H7) for any $X \in P(U)$. Moreover, if $\underline{apr}$ and $\overline{apr}$ satisfy (H7) for any $X \in P(U)$, then $\underline{apr}$ and $\overline{apr}$ satisfy (L7) for any $X \in P(U)$.*
(12) *If $\underline{apr}$ and $\overline{apr}$ satisfy (L7) or (H7) for any $X \in P(U)$, then*

$$\text{(L8)} \Longleftrightarrow \text{(H8)}, \ \text{(L8′)} \Longleftrightarrow \text{(H8′)}, \ \text{(L9)} \Longleftrightarrow \text{(H9)}, \ \text{(L9′)} \Longleftrightarrow \text{(H9′)}.$$

(13) *If $\underline{apr}$ satisfies (L1) and (L8′) for any $X \in P(U)$, then $\underline{apr}$ satisfies (L8) for any $X \in P(U)$.*
(14) *If $\overline{apr}$ satisfies (H1) and (H8′) for any $X \in P(U)$, then $\overline{apr}$ satisfies (H8) for any $X \in P(U)$.*
(15) *If $\underline{apr}$ satisfies (L1) for any $X \in P(U)$, then $\underline{apr}$ satisfies (L9′) for any $X \in P(U)$.*
(16) *If $\overline{apr}$ satisfies (H1) for any $X \in P(U)$, then $\overline{apr}$ satisfies (H9′) for any $X \in P(U)$.*