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A comparative study of multigranulation rough sets and concept lattices via rule acquisition

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ABSTRACT

Recently, by combining rough set theory with granular computing, pessimistic and optimistic multigranulation rough sets have been proposed to derive "AND" and "OR" decision rules from decision systems. At the same time, by integrating granular computing and formal concept analysis, Wille's concept lattice and object-oriented concept lattice were used to obtain granular rules and disjunctive rules from formal decision contexts. So, the problem of rule acquisition can bring rough set theory, granular computing and formal concept analysis together. In this study, to shed some light on the comparison and combination of rough set theory, granular computing and formal concept analysis, we investigate the relationship between multigranulation rough sets and concept lattices via rule acquisition. Some interesting results are obtained in this paper: (1) "AND" decision rules in pessimistic multigranulation rough sets are proved to be granular rules in concept lattices, but the inverse may not be true; (2) the combination of the truth parts of an "OR" decision rule in optimistic multigranulation rough sets is an item of the decomposition of a disjunctive rule in concept lattices; (3) a non-redundant disjunctive rule in concept lattices is shown to be the multi-combination of the truth parts of "OR" decision rules in optimistic multigranulation rough sets; and (4) the same rule is defined with a same certainty factor but a different support factor in multigranulation rough sets and concept lattices. Moreover, algorithm complexity analysis is made for the acquisition of "AND" decision rules, "OR" decision rules, granular rules and disjunctive rules.

1. Introduction

Rough set theory, presented by Pawlak [42], has drawn many attentions from researchers over the past thirty-three years [20,26,43,78,85,86]. As is well known, its original idea is to partition the universe of discourse into disjoint subsets by a given equivalence or indiscernibility relation. Furthermore, these obtained disjoint subsets are viewed as the basic knowledge which is used to characterize any target set by means of the so-called lower and upper approximations.

Since the equivalence or indiscernibility relation has its limitations in dealing with *information systems* with fuzzy, continuous-valued or interval-valued attributes, the classical rough sets have been generalized and developed by some scholars [14,16,18,57,59,73,77]. Note that the generalized and developed rough sets are beneficial to the implementation of rule acquisition in different kinds of *decision systems* [6,8,11,24,27,88,89].

From the aspect of *granular computing* presented by Zadeh [83] and further elaborated by other researchers [2,44,45,52,74], the aforementioned generalized and developed rough sets describe a target set by the lower and upper approximations under one granulation. However, in the real world, multiple granulations are sometimes required to approximate a target set as well. For example, multi-scale data sets need multiple granulations for set approximations [66], and multi-source data sets inspire Qian et al. [48,49] to put forward pessimistic multigranulation rough sets and optimistic multigranulation rough sets for applying multi-source information fusion. These information fusion strategies were soon extended to the cases of incomplete, neighborhood, covering and fuzzy environments [17,36,37,55,68,71]. Moreover, it deserves to be mentioned that the pessimistic and optimistic multigranulation rough sets were used in [48,49] to derive

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"AND" and "OR" decision rules from decision systems, which was further discussed by Yang et al. [70] in terms of local and global measurements of the "AND" and "OR" decision rules.

Formal concept analysis, presented by Wille [64] in the same year as rough set theory, has attracted many researchers [4,56,60,84,87] to this promising field. Up to now, its applications cover data mining [1,13], knowledge discovery [10,46,69], machine learning [23], software engineering [53], etc. Within this theory, *formal contexts*, *formal concepts* and *concept lattices* are three basic notions for data analysis.

Also, it deserves to be mentioned that in recent years both multigranulation rough sets and concept lattices have been connected with *three-way decisions* whose unified framework description and superiority were given by Yao [78–80] and whose further investigations and applications were studied by many scholars [9,15,19,28,35,39,72,82]. For example, Qian et al. [50] established a new decision-theoretic rough set [81] from the perspective of multigranulation rough sets. By taking the intension part of a formal concept as an orthopair [7], Qi et al. [47] put forward *three-way concept lattice* and discussed some useful properties. In addition, Li et al. [30] proposed another three-way concept lattice with the name of *approximate concept lattice* under the environment of incomplete data, where the intension part of a formal concept was expressed as a nested pair which is in fact equivalent to an orthopair.

Recently, more and more attention [12,21,22,25,54,62,75] has been paid to comparing and combining rough set theory and formal concept analysis. Under such a circumstance, object-oriented concept lattice was introduced in [76] by incorporating lower and upper approximation ideas into concept-forming operators and it was further elaborated in [41,61]. Ren et al. [51] presented the notion of a disjunctive rule in *formal decision contexts* [86] by the help of Wille's and object-oriented concept lattices. Moreover, covering-based rough sets and concept lattices were related to each other in [5,58] from the viewpoints of approximation operators and reduction. In the meanwhile, integrating formal concept analysis with granular computing has also attracted many researchers [40,63]. For instance, Wu et al. [65] put forward the notion of a granular rule in formal decision contexts. Li et al. [29] discussed the relation between granular rules and *decision rules* [31]. In addition, rough set theory has been related to granular computing [48,49,66,74], and vice versa.

What is more, the comparison and combination of rough set theory, granular computing and formal concept analysis has received much attention in knowledge representation and discovery [32,65,67,75]. The main contributions of the existing literature in this aspect can be summarized as follows: (1) rough set theory, granular computing and formal concept analysis were combined to form composite concept-forming operators [75] and induce granular concepts [65]; and (2) they were jointly used to recognize cognitive concepts by two-step learning approaches [32,67]. However, little attention has been paid to comparing and combining these three theories from the perspective of rule acquisition. This problem deserves to be investigated since it can not only shed some light on the comparison and combination of them, but also help us to make better decision analysis of the data.

Motivated by the above problem, this study mainly focuses on the comparison and combination of rough set theory, granular computing and formal concept analysis from the viewpoint of rule acquisition. More specifically, we put forward an effective way of transforming decision systems into formal decision contexts and discuss some useful properties. And then the relationship between multigranulation rough sets and concept lattices is analyzed from the perspectives of differences and relations between rules, support and certainty factors for rules, and algorithm complexity analysis of rule acquisition.

The rest of this paper is organized as follows. In Section 2, we recall the notions of Pawlak's rough set, pessimistic multigranulation rough set and optimistic multigranulation rough set as well as their induced "AND" and "OR" decision rules. Moreover, Wille's concept lattice, object-oriented concept lattice, granular rules and disjunctive rules are introduced. In Section 3, we transform decision systems into formal decision contexts and discuss some useful properties. In Section 4, the relationship between multigranulation rough sets and concept lattices is analyzed from the viewpoints of differences and relations between rules, support and certainty factors for rules, and algorithm complexity analysis of the acquisition of "AND" decision rules, "OR" decision rules, granular rules and disjunctive rules. Section 5 concludes this paper with a brief summary and an outlook for further research.

2. Preliminaries

In this section, we review some basic notions such as information system, Pawlak's rough set, pessimistic multigranulation rough sets, "AND" decision rules, optimistic multigranulation rough sets, "OR" decision rules, formal context, Wille's concept lattice, object-oriented concept lattice, granular rules and disjunctive rules.

2.1. Pawlak's rough set

Let U be a non-empty finite set of objects and AT be a non-empty finite set of attributes. Then an information system is considered as a pair $S=(U,AT)$ [42], where the value of $x\in U$ under attribute $a\in AT$ is denoted by $a(x)$.

Given $A\subseteq AT$, an equivalence relation $IND(A)$ is defined as

$$IND(A)=\{(x,y)\in U\times U:\forall a\in A, a(x)=a(y)\}, \tag{1}$$

which partitions U into equivalence classes $[x]_A=\{y\in U:(x,y)\in IND(A)\}$. This partition $\{[x]_A:x\in U\}$ is often denoted by $U/IND(A)$. For a subset X of U,

$$\underline{A}(X)=\{x\in U:[x]_A\subseteq X\} \text{ and } \overline{A}(X)=\{x\in U:[x]_A\cap X\neq\emptyset\} \tag{2}$$

are called the lower and upper approximations [42], respectively. The ordered pair $[\underline{A}(X),\overline{A}(X)]$ is said to be Pawlak's rough set of X with respect to A.

2.2. Multigranulation rough sets and their induced rules

Different from Pawlak's rough set model, multigranulation rough sets were established on the basis of a family of equivalence relations rather than a single one only.

Definition 1 [48]**.** Let S be an information system and $A_1,A_2,\ldots,A_s\subseteq AT$. Then the pessimistic multigranulation lower and upper approximations of a subset X of U are respectively defined as

$$\underline{\sum_{j=1}^{s}A_j^P}(X)=\{x\in U:[x]_{A_1}\subseteq X\wedge[x]_{A_2}\subseteq X\wedge\cdots\wedge[x]_{A_s}\subseteq X\}$$
$$\text{and } \overline{\sum_{j=1}^{s}A_j^P}(X)=\sim\underline{\sum_{j=1}^{s}A_j^P}(\sim X), \tag{3}$$

where $[x]_{A_j}$ $(1\le j\le s)$ is the equivalence class of x in terms of A_j, $\wedge$ is the logical conjunction operator, and $\sim X$ is the complement of X with respect to U.