

# Double-quantitative fusion of accuracy and importance: Systematic measure mining, benign integration construction, hierarchical attribute reduction



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## ABSTRACT

Uncertainty measure mining and applications are fundamental, and it is possible for double-quantitative fusion to acquire benign measures via heterogeneity and complementarity. This paper investigates the double-quantitative fusion of relative accuracy and absolute importance to provide systematic measure mining, benign integration construction, and hierarchical attribute reduction. (1) First, three-way probabilities and measures are analyzed. Thus, the accuracy and importance are systematically extracted, and both are further fused into importance-accuracy (IP-Accuracy), a synthetic causality measure. (2) By sum integration, IP-Accuracy gains a bottom-top granulation construction and granular hierarchical structure. IP-Accuracy holds benign granulation monotonicity at both the knowledge concept and classification levels. (3) IP-Accuracy attribute reduction is explored based on decision tables. A hierarchical reduct system is thereby established, including qualitative/quantitative reducts, tolerant/approximate reducts, reduct hierarchies, and heuristic algorithms. Herein, the innovative tolerant and approximate reducts quantitatively approach/expand/weaken the ideal qualitative reduct. (4) Finally, a decision table example is provided for illustration. This paper performs double-quantitative fusion of causality measures to systematically mine IP-Accuracy, and this measure benignly constructs a granular computing platform and hierarchical reduct system. By resorting to a monotonous uncertainty measure, this study provides an integration-evolution strategy of granular construction for attribute reduction.

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## 1. Introduction

*Rough set theory (RS-Theory)* [35,36,59,63,70,71] represents a fundamental *granular computing (GrC)* pattern for handling uncertainty issues. The initial Pawlak-Model [35] acts only as a qualitative model, so it lacks the quantitative mechanism regarding fault-tolerance and robustness. Thus, quantitative models exhibit improvements and have applications, and they could in part be unified by the subthreshold measure [61]. In particular, the *probabilistic rough set (PRS)* [1,27,28,30,51,57,58,60,78] introduces the probability uncertainty measure into RS-Theory, which forms the basis of mainstream quantitative models. PRS offers measurability, generality, and flexibility and exhibits a series of concrete models, including the *decision-theoretic rough set (DTRS)* [64],

game-theoretic rough set [1,2], variable precision rough set [81], Bayesian rough set [47], and parameterized rough set [6]. With the exception of PRS, the graded rough set [25,62] depends on the grade measure to become another basic type of quantitative model.

Herein, DTRS is introduced as a model example. DTRS utilizes conditional probability and the Bayesian risk decision to establish three-way decisions and threshold-quantitative semantics [64]. As a result, DTRS improves upon some basic models and provides a quantitative exploration platform. In terms of relevant studies, three-way decisions were analyzed in [17–19,58,60,79]; model development and threshold calculation were discussed in [15,16,27,45,48]; attribute reduction was studied in [10,12,31,65,74,75]; and model applications (regarding clustering, regression, and semi-supervised learning) were addressed in [13,23,24,26,66]. In fact, three-way decisions have been expanded into three-way decision theory, and this fundamental theory has been the subject of extensive study and used in a number of

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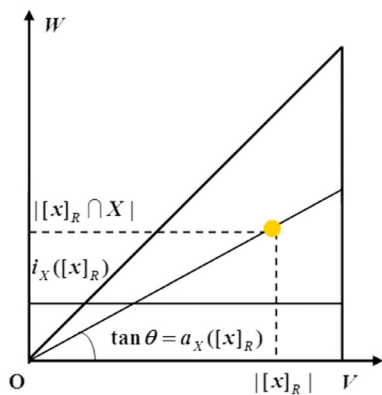


Fig. 1. Accuracy and importance in the cardinality plane.

Table 1  
Main abbreviations of this paper.

Abbreviation	Original term
<i>RS-Theory</i>	Rough set theory
<i>GrC</i>	Granular computing
<i>DTRS</i>	Decision-theoretic rough set
<i>Approx-Space</i>	Approximate space
<i>C-POS/C-BND</i>	Classification-positive/boundary region
<i>POS/NEG/BND</i>	Set-positive/negative/boundary region
<i>D-Table</i>	Decision table
<i>Kn-Coarsening</i>	Knowledge coarsening
<i>Gr-Merging</i>	Granule merging
<i>Gr-Preservation</i>	Granule preservation
<i>IP-Accuracy</i>	Importance-accuracy
<i>IIP-Accuracy</i>	Internal importance-accuracy
<i>Gr/Kn/Cl/Con</i>	Granule/knowledge/classification/concept
<i>Gr-Con/Kn-Con/</i> <i>Kn-Cl</i>	Granular concept/knowledge's concept/knowledge's classification
<i>MT</i>	Monotonicity target

Table 2  
Three-way probabilities and three-way measures in Bottom-System (*U, R, X*).

Metrical essence	Causality relevance	Metrical features	Three-way probabilities	Probability formula	Three-way measures	Measure formula
$\frac{ [x]_R \cap X }{ [x]_R }$	Has	Relativity, concentration, locality	Likelihood probability	$p(X [x]_R)$	Accuracy	$a_X([x]_R)$
$\frac{ [x]_R \cap X }{ X }$	Has	Absoluteness, directness, globality (regarding <i>X</i> )	Posterior probability	$p([x]_R X)$	Importance	$i_X([x]_R)$
$\frac{ [x]_R }{ U }$	Never has	Absoluteness, vividness, globality (regarding <i>U</i> )	Prior probability	$p([x]_R)$	Generality	$g([x]_R)$

Table 3  
Granule-statistical information in Bottom-System (*U, C, X*).

Granule $[x]_C^i$	Interaction cardinality $ [x]_C^i \cap X $	Granule cardinality $ [x]_C^i $	Generality $g([x]_C^i)$	Importance $i_X([x]_C^i)$	Accuracy $a_X([x]_C^i)$	IP-Accuracy $ia_X([x]_C^i)$
$[x]_C^1$	8	16	0.16	0.200	0.5	0.1
$[x]_C^2$	6	10	0.10	0.150	0.6	0.09
$[x]_C^3$	6	15	0.15	0.150	0.4	0.06
$[x]_C^4$	5	10	0.10	0.125	0.5	0.0625
$[x]_C^5$	4	16	0.16	0.100	0.25	0.025
$[x]_C^6$	3	3	0.03	0.075	1.0	0.075
$[x]_C^7$	3	4	0.04	0.075	0.75	0.05625
$[x]_C^8$	2	3	0.03	0.050	2/3	1/30
$[x]_C^9$	2	6	0.06	0.050	1/3	1/60
$[x]_C^{10}$	1	1	0.01	0.025	1.0	0.025
$[x]_C^{11}$	0	8	0.08	0.000	0.0	0.0
$[x]_C^{12}$	0	8	0.08	0.000	0.0	0.0

Table 4  
Knowledge's relevant uncertainty measures regarding IP-Accuracy.

Knowledge <i>C</i> <sub>*</sub>	Kn-Con IP-Accuracy <i>ia</i> <sub>X</sub> ( <i>C</i> <sub>*</sub> ), <i>ia</i> <sub>−X</sub> ( <i>C</i> <sub>*</sub> )	Kn-Con IIP-Accuracy <i>lia</i> <sub>X</sub> ( <i>C</i> <sub>*</sub> ), <i>lia</i> <sub>−X</sub> ( <i>C</i> <sub>*</sub> )	Kn-Cl IP-Accuracy <i>ia</i> <sub>D</sub> ( <i>C</i> <sub>*</sub> )	Kn-Cl IIP-Accuracy <i>lia</i> <sub>D</sub> ( <i>C</i> <sub>*</sub> )	IP-Accuracy discrepancy, equality rates <i>dr</i> <sub><i>ia</i><sub>D</sub></sub> , <i>er</i> <sub><i>ia</i><sub>D</sub></sub>	IIP-Accuracy discrepancy, equality rates <i>dr</i> <sub><i>lia</i><sub>D</sub></sub> , <i>er</i> <sub><i>lia</i><sub>D</sub></sub>
{ <i>a, b, c, d, e, f</i> }	0.5437, 0.6958	0.1000, 0.2667	1.2395	0.3667	0.00%, 100.00%	0.00%, 100.00%
{ <i>a, b, c, d, e</i> }	0.5437, 0.6958	0.1000, 0.2667	1.2395	0.3667	0.00%, 100.00%	0.00%, 100.00%
{ <i>a, b, c, d</i> }	0.5376, 0.6917	0.1000, 0.2667	1.2293	0.3667	0.82%, 99.18%	0.00%, 100.00%
{ <i>a, b, c</i> }	0.5366, 0.6910	0.1000, 0.2667	1.2276	0.3667	0.96%, 99.04%	0.00%, 100.00%
{ <i>a, b</i> }	0.5193, 0.6796	0.1000, 0.0000	1.1989	0.1000	3.28%, 96.72%	72.73%, 27.27%
{ <i>a, c</i> }	0.5050, 0.6700	0.1000, 0.2667	1.1750	0.3667	5.20%, 94.80%	0.00%, 100.00%
{ <i>b, c</i> }	0.5314, 0.6876	0.0000, 0.2667	1.2190	0.1000	1.65%, 98.35%	27.27%, 72.73%
{ <i>a</i> }	0.4375, 0.6250	0.1000, 0.0000	1.0625	0.1000	14.28%, 85.72%	72.73%, 27.27%
{ <i>b</i> }	0.5141, 0.6761	0.0000, 0.0000	1.1902	0.1000	3.98%, 96.02%	100.00%, 0.00%
{ <i>c</i> }	0.4762, 0.6508	0.0000, 0.2667	1.1270	0.1000	9.08%, 90.92%	27.27%, 72.73%
∅	0.4000, 0.6000	0.0000, 0.0000	1.0000	0.0000	19.32%, 80.68%	100.00%, 0.00%

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