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A comprehensive review of multi criteria decision making approaches based on interval type-2 fuzzy sets

Erkan Celik *, Muhammet Gul, Nezir Aydin, Alev Taskin Gumus, Ali Fuat Guneri

Department of Industrial Engineering, Yildiz Technical University, 34349 Istanbul, Turkey

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ABSTRACT

Multi criteria decision making (MCDM) is a discipline of operations research which has widely studied by researchers and practitioners. It deals with evaluating and ranking alternatives from the best to the worst under conflicting criteria with respect to decision maker(s) preferences. Since, many real-world systems include uncertainty and vagueness in information, MCDM uses fuzzy sets. In recent years, as an extension of the traditional fuzzy sets concept, type-2 fuzzy sets are preferred to have the capability of handling more uncertainty, and hence, to produce more accurate and robust results, MCDM approaches based on interval type-2 fuzzy sets (IT2FSs) have been published in various subjects. This paper reviews 82 different papers using various MCDM approaches based on IT2FSs which are classified into 35 categories. All papers with respect to single and hybrid approaches are discussed, pointing out their real applications or empirical results and limitations. Furthermore, the papers are statistically analyzed to show new trends within the context of IT2FSs. This systematic and comprehensive review study provides an insight for researchers on interval type-2 fuzzy MCDM in terms of showing current state and potential areas to be focused in the future.

1. Introduction

MCDM is a discipline of operations research which has been widely studied by researchers and practitioners [60,7,6]. It concerns with evaluating, assessing and selecting alternatives from the best to the worst under conflicting criteria with respect to decision maker(s) preferences. The main characteristics of a MCDM method include: (1) the alternatives to be evaluated, (2) the criteria against which the alternatives are evaluated, (3) scores that reflect the value of an alternative's expected performance on the criteria, and (4) criteria weights that measure the relative importance of each criterion as compared with others [5]. It is common that linguistic variables are used to assess the ratings of various criteria and alternatives in real-world problems [16]. The concept of linguistic variables is useful in dealing with complex situations. Linguistic values are frequently represented by fuzzy sets [36] and it makes more sense than exact numbers [93]. Thereby, fuzzy multi criteria decision making (FMCDM) discipline is emerged as an area that incorporates fuzzy sets and MCDM problems. On the other hand, the superiority of fuzzy sets has been varying for different types of fuzzy models such as type-1, type-2 and higher types because they are designed to meet varying levels of uncertainty [38]. IT2FSs are more suitable to reflect uncertainties [20] for applying in fuzzy decision making problems [66,51].

The fundamental of a type-2 fuzzy set was presented by Zadeh [93] as an extension of the concept of a type-1 fuzzy set [56,62]. An interval type-2 fuzzy set is a special version of a general type-2 fuzzy set. Since applying general type-2 fuzzy systems contains complex computational operations [66], it has not widely applied to real world applications [55]. IT2FSs are the most frequently used type-2 fuzzy sets [63] because of their easiness and reduced computational effort in comparison with general type-2 fuzzy sets. It is also emphasized that type-2 fuzzy sets handle more uncertainty [65], and hence, produces more accurate and robust results [38,37]. Therefore, it is widely used in various application areas [64,38,51]. Dereli et al. [38] presented a concise literature review of industrial applications of type-2 fuzzy sets and systems. Castillo and Melin [8] and Castillo and Melin [9] presented reviews about the design and optimization of interval type-2 fuzzy controllers, and optimization of type-2 fuzzy systems based on bio-inspired methods, respectively. Melin and Castillo [61] also presented applications of type-2 fuzzy logic in classification and pattern recognition. Unfortunately, no systematic and comprehensive review has been provided for MCDM approaches based on IT2FSs. However, there have been several MCDM approaches based

* Corresponding author. Tel.: +90 212 383 2998; fax: +90 212 258 5928.
E-mail address: erkcelik@yildiz.edu.tr (E. Celik).

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Table 1
Brief explanation of MCDM approaches.

Abbreviation	Method	Description
IT2FAHP	Interval type-2 fuzzy analytic hierarchy process	A structured MCDM technique with pairwise comparison manner by considering concept of IT2FSs
IT2FTOPSIS	Interval type-2 fuzzy technique for order of preference by similarity to ideal solution	A MCDM technique based on the concept of choosing the solution with the shortest distance from the ideal solution and the farthest distance from the negative ideal solution by considering concept of IT2FSs
IT2FPROMETHEE	Interval type-2 fuzzy preference ranking organization method for enrichment of evaluations	An outranking method based on a pairwise comparison of alternatives for each recognized criterion within the environment of IT2FSs for MCDM
IT2FQUALIFLEX	Interval type-2 fuzzy qualitative flexible multiple criteria method	An outranking method based on testing how each possible ranking of alternatives is supported by different criteria within the environment of IT2FSs for MCDM
IT2FDEMATEL	Interval type-2 fuzzy decision-making trail and evaluation laboratory	A MCDM technique considering the causal relationships between criteria and illustrating the weights between criteria by diagraphs within the environment of IT2FSs
IT2FCOPRAS	Interval type-2 fuzzy complex proportional assessment	A compromising MCDM technique aimed to rank a set of alternatives according to their significance and utility degree by means of IT2FSs
IT2FDEA	Interval type-2 fuzzy data envelopment analysis	A method for measuring the relative efficiency of a set of homogeneous decision-making units with type-2 fuzzy inputs and outputs
IT2FELECTRE	Interval type-2 fuzzy Elimination et choix traduisant la realité	An outranking method based on pairwise comparisons of the alternatives to determine the concordance and discordance sets considering the context of IT2FSs
IT2FLINMAP	Interval type-2 fuzzy linear programming technique for multi-dimensional analysis of preference	An optimization based MCDM method for determining the optimal weight of the criteria and generating the best compromise alternative with respect to pairwise comparisons of alternatives by applying concept of IT2FSs
IT2FLA	Interval type-2 fuzzy linear assignment	A linear compensatory process for the interaction and combination of the criteria by applying the concept of IT2FSs based on signed distances
TIT2FFWA	Interval type-2 fuzzy Frank weighted averaging	A MCDM approach by using Frank weighted averaging and weighted geometric operator based on IT2FSs
IT2FGRA	Interval type-2 fuzzy gray relational analysis	A compromise ranking approach based on the gray system theory, which can be used to solve complicated inter-relationships among multiple performance by using IT2FSs
IT2FVIKOR	Interval type-2 fuzzy visekriterijumska optimizacija i kompromisno resenje	Method for determining the compromise ranking-list of a set of alternatives according to the measure of closeness to the ideal solution by using IT2FSs
IT2FIA	Interval type-2 fuzzy information axiom	A method that states the design which has the smallest information content is the best design among those designs that satisfy the first axiom of axiomatic design
HFTOPSIS	Hesitant fuzzy technique for order of preference by similarity to ideal solution	A compromise ranking approach based on closeness coefficient by means of the hesitant fuzzy linguistic terms
MCGP	Multi-choice goal programming	An approach which allows decision makers to set multi choice aspiration levels for each goal
SAW	Simple additive weighting	A MCDM approach that aimed to determine a weighted score for each alternative by adding the contributions of each attribute multiplied by their weights

on IT2FSs (Table 1). Therefore, in this paper, we review the literature related to the approaches to MCDM based on IT2FSs using academic databases of ScienceDirect, Springer, Taylor&Francis, Wiley, SagePub, Emerald, ProQuest, IEEE Xplore, and Hindawi. A total of 82 papers were reviewed ranged from 2007 to April 2015. Our contributions to the literature are as follows: (1) to determine the MCDM approaches that have been integrated with IT2FSs, (2) to reveal application areas (education, energy, environment, healthcare, human resource management (HRM), investment management, manufacturing, risk management, technology management, transportation and logistics) that have further used these MCDM approaches based on IT2FSs, (3) to show the countries which published papers related to these MCDM approaches based on IT2FSs, and (4) to determine how the trend of type-2 fuzzy MCDM will continue in the future.

The remainder of this paper is organized as follows. Section 2 provides a brief overview on the fundamentals of type-2 fuzzy sets. Section 3 and 4 present the single and hybrid approaches, respectively. Section 5 provides discussion of the review based statistical analysis results of the review. Finally, Section 6 presents conclusion, limitations, and recommendations for future studies.

2. Interval type-2 fuzzy sets

In this section, we briefly review some definitions of type-2 fuzzy sets and IT2FSs from Mendel et al. [66], Lee and Chen [59], Chen and Lee [20], Celik et al. [13,12], Kahraman et al. [55]:

Definition 1. A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows:

$$\tilde{\tilde{A}} = \left\{((x,u), \mu_{\tilde{\tilde{A}}}(x,u)) | \forall x \in X, \forall u \in J_X \subseteq [0,1], 0 \leqslant \mu_{\tilde{\tilde{A}}}(x,u) \leqslant 1\right\}$$

where J_X denotes an interval in $[0,1]$. Moreover, the type-2 fuzzy set $\tilde{\tilde{A}}$ also can be represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_X} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u)$$

where $J_X \subseteq [0,1]$ and $\iint$ denotes union over all admissible x and u.

Definition 2. Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called as an interval type-2 fuzzy set. An interval type-2 fuzzy set $\tilde{\tilde{A}}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_X} 1/(x,u) \text{ where } J_X \subseteq [0,1].$$

Definition 3. The upper membership function and the lower membership function of an interval type-2 fuzzy set are type-1 membership functions, respectively. The reference points and the heights of the upper and the lower membership functions of IT2FSs are used to characterize IT2FSs by Chen and Lee [20]. Fig. 1 shows a trapezoidal interval type-2 fuzzy set $\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = \left(\left(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1\left(\tilde{A}_i^U\right), H_2\left(\tilde{A}_i^U\right)\right), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L;\right.$