



# Optimization of a multiproduct economic production quantity problem with stochastic constraints using sequential quadratic programming



Seyed Hamid Reza Pasandideh <sup>a,1</sup>, Seyed Taghi Akhavan Niaki <sup>b,\*</sup>, Abolfazl Gharaei <sup>a,1</sup>

<sup>a</sup> Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran

<sup>b</sup> Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

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## ABSTRACT

In this paper, a multiproduct single vendor–single buyer supply chain problem is investigated based on the economic production quantity model developed for the buyer to minimize the inventory cost. The model to be more applicable for real-world supply chain problems contains five stochastic constraints including backordering cost, space, ordering, procurement, and available budget. The objective is to find the optimal order quantities of the products such that the total inventory cost is minimized while the constraints are satisfied. The recently-developed sequential quadratic programming (SQP), as one of the best optimization methods available in the literature, is used to solve the problem. Twenty numerical examples in 3 scales of small, medium, and large are solved in order to demonstrate the applicability of the proposed methodology and to evaluate its optimum performance. The results show that SQP has satisfactory performance in terms of optimum solutions, number of iterations to achieve the optimum solution, infeasibility, optimality error, and complementarity. Besides, the optimum performance of the SQP method is compared with the one of another exact method called interior point using the above numerical examples under similar conditions. The comparison results are in favor of the employed SQP. At the end, a sensitivity analysis is performed on the change rate of the objective function obtained based on the change rate of the variance of the order quantity.

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## 1. Introduction and research literature

The economic production quantity (EPQ) model is often used in the manufacturing sector to assist firms in determining the optimal production lot size that minimizes overall production–inventory costs. Among an extensive research conducted on the applications of the EPQ model one can mention the following recent works in chronological order.

In the first decade of 2000, Jamal et al. [1] developed an EPQ model to determine the optimum batch quantity in a single-stage system. They assumed rework to be completed under two different operational policies named immediate and N-Cycle rework processes, to minimize the total system cost. Huang [16] introduced an EPQ model in which imperfect products were allowed into the produced lot sizes. Chiu et al. [8] considered the effects of random defective rate and imperfect rework process on the EPQ model. However, they did not take into account the service level as an

operational performance. Chang [6] investigated the effects of imperfect products on the total inventory cost associated with an EPQ model. Goyal and Cárdenas-Barrón [14] presented an EPQ model by considering an imperfect production system that would produce defective products, supposing all the defective items that were randomly produced were reworked. Chiu et al. [9] investigated an EPQ model with scrap, rework, and stochastic machine breakdowns to determine the optimal run time and production quantity. Choi et al. [10] developed a production model in which demand was satisfied by recovering used products as well as new products. They assumed that a fixed proportion of the used products were collected from customers and later recovered for reuse. Sarker et al. [27] introduced an extended EPQ model dealing with the optimum production quantity in a multi-stage system in which rework was performed under two different operational policies to minimize the total system cost. Jaber et al. [17] developed an EPQ model for items with imperfect quality subject to learning effects. They assumed imperfect quality items were withdrawn from inventory and sold at a discount price. However, they did not consider the learning effect as it can affect production rate and shortage costs, directly. Cárdenas-Barrón [3] proposed a simple derivation to find the optimal production quantity of a system

\* Corresponding author. Tel.: +98 21 66165740; fax: +98 21 66022702.

E-mail addresses: [shr\\_pasandideh@khu.ac.ir](mailto:shr_pasandideh@khu.ac.ir) (S.H.R. Pasandideh), [Niaki@Sharif.edu](mailto:Niaki@Sharif.edu) (S.T.A. Niaki), [ab.gharaei@gmail.com](mailto:ab.gharaei@gmail.com) (A. Gharaei).

<sup>1</sup> Tel.: +98 (21) 88830891; fax: +98 (21) 88329213.

that includes a rework process. Liao et al. [18] studied an integrated maintenance and production system with an EPQ model for an imperfect process involving a defective production system under increasing failure rate. Yoo et al. [31] proposed a profit-maximizing EPQ model that incorporates both the imperfect production quality and two-way imperfect inspection. Widyadanaa and Wee [30] provided a multi-product EPQ vendor–buyer integrated model in just-in-time philosophy under budget constraint. The problem was modeled as a mixed integer non-linear program (MINLP) with constraint. Cárdenas-Barrón [4] introduced an EPQ model with a rework process at a single-stage manufacturing system with planned backorders. Some of the other works in this decade include Chandrasekaran et al. [7], Liu et al. [19], Mohan et al. [20], Cárdenas-Barrón [5], and Parveen and Rao [22].

In the second decade of 2000, Pasandideh et al. [24] developed a multi-product EPQ model in which there were some imperfect items of different types being produced such that reworks were allowed and that there was a warehouse space limitation. Under these conditions, they formulated the problem as a nonlinear integer-programming model and proposed a genetic algorithm to solve it. Taleizadeh et al. [28] introduced an EPQ model with scrapped items and limited production capacity in a multiproduct single-machine production system with stochastic scrapped production rate, partial backordering, and service level constraint. They did not consider the impact of rejection and rework in their model. Furthermore, they only considered a limited production capacity. Taleizadeh et al. [29] studied two joint production systems in the form of a multiproduct single machine with and without rework. In their work, shortage was allowed and backordered. For each system, the optimal cycle length and the backordered and production quantities of each product were determined such that the cost function is minimized. The results obtained by solving the models with and without rework of defective items along with sensitivity analysis and some managerial insights based on the numerical illustration were provided in their research. Hafshejani et al. [15] developed a multi-product EPQ model under warehouse space limitation by assuming that each produced lot would contain some imperfect items and scraps. Under these conditions, they formulated the problem as a non-linear programming model and proposed a genetic algorithm to solve it. Although some constraints including a limitation on the available procurement budget was considered in their work, backorder and ordering costs were not taken into account in their modeling. Besides, rework and its impact have not been considered either. Pasandideh et al. [25] presented a multi-item multi-period inventory control problem with all-unit and/or incremental quantity discount policies under limited storage capacity. In their work, the independent random demand rates of the items in the periods were known and the items were supplied in distinct batch sizes. The objective was to find the optimal order quantities of all items in different periods such that the total inventory cost would be minimized and the constraint was satisfied. A mixed binary integer programming model was first developed in this work to model the problem and then a parameter-tuned genetic algorithm (GA) was employed to solve it. Mousavi et al. [21] modeled a seasonal multi-product multi-period inventory control problem. In their research, inventory costs were obtained under inflation and all-unit discount policy. Furthermore, the products were delivered in boxes of known number of items, and in case of shortage, a fraction of demand was considered backorder and a fraction lost sale. Besides, the total storage space and total available budget were limited. The objective of their work was to find the optimal number of boxes of the products in different periods to minimize the total inventory cost. As the integer nonlinear model of their problem was hard to solve using exact methods, a particle swarm optimization (PSO)

algorithm was proposed to find a near-optimal solution. They did not consider the service level constraint in their model. Besides, despite the difficulty involved in their model, exact methods were good alternative solution approaches that were not taken into account. Pasandideh et al. [23] investigated the vendor managed inventory (VMI) problem of a single vendor–single buyer supply chain, in which the vendor was responsible to manage the buyer's inventory.

To suit real-world inventory problems, in this paper, the EPQ model of the single vendor–single buyer supply chain problem proposed by Pasandideh et al. [23] is extended to include five new stochastic constraints of (1) backorder cost, (2) space, (3) ordering, (4) procurement, and (5) available budget. The main objective here is to find the order quantities of the products such that the stochastic constraints are satisfied and the total inventory cost is minimized. To answer this main research question, SQP is used as a technique to solve the derived nonlinear programming problem (NLP). This iterative procedure that models an NLP for a given iterate using a quadratic programming (QP) sub-problem, first solves that QP subproblem, and then uses the solution to construct a new iterate. SQP is a general exact method to solve nonlinear optimization problems. Although it has been employed to solve some nonlinear optimization problems previously, to the best of authors' knowledge, it has never been used to solve inventory control problems with stochastic constraints.

The outline of the rest of the paper is as follows: Section 2 sheds light on the problem definition and assumptions. The model derivation is described in Section 3. Section 4 deals with the sequential quadratic programming method. Numerical examples in 3 scales of small, medium, and large are presented in Section 5. In Section 6, the optimum performance of the sequential quadratic programming method is compared with the one of another exact solution method called the interior point. Finally, sensitivity analysis and conclusion are presented in Sections 7 and 8, respectively.

## 2. Problem definition and assumptions

The problem at hand arises from a single-buyer inventory control environment that uses EPQ system to minimize total inventory costs for all products. In this problem, there are several products and the EPQ model is utilized with practical instances of finite production rate, limited procurement cost, limited backorder cost, limited space cost, limited ordering cost, and limited available budget for procurement, backorder, space and ordering. The objective is to find the order quantities of the products such that the total inventory cost is minimized while the stochastic constraints are satisfied. The following assumptions are used for formulation of the problem:

- (1) There is a single buyer to minimize his (her) total inventory cost.
- (2) There are  $n$  products.
- (3) The planning horizon is infinite.
- (4) The lead time is assumed negligible.
- (5) Selling prices of all products are fixed.
- (6) Quantity discount is not allowed.
- (7) Production rate for all products is continuous and finite.
- (8) For each product, shortage is allowed and backordered ( $\hat{\pi} \neq 0$  and  $\pi = 0$ ).
- (9) Buyer's demand rate for all products is known and constant.
- (10) The total backorder cost of all products is less than  $B$  with a probability greater than  $\alpha$ .
- (11) The total space cost of all products is less than  $S$  with a probability greater than  $\alpha$ .

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