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Building the meaning of preference from logical paired structures

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ABSTRACT

Making decisions by learning preferences requires to consider semantical aspects dealing with the meaning and use of the preference concept. Examining recent developments on bipolarity, where concepts are measured/verified regarding a pair of opposite poles, we focus on the dialectic process by which the meaning of concepts emerges. Our proposal is based on the *neutrality* in between the opposite poles, such that a basic type of structure is used to characterize in logical terms the concepts and the knowledge that they generate. In this paper we model the meaning of concepts by *paired structures*, and apply these structures for learning and building the different meanings of *preference* for decision making.

1. Introduction

Concepts allow understanding the multiple stimuli and perceptions coming from reality, organizing and giving sense to all the relevant information surrounding us. In this sense, the meaning/use of concepts can be verified by data, based on a valuation structure specifically designed for such verification process.

In psychology (see e.g. [9,29,37]), the meaning of concepts is generally developed on a semantic theory which became a corner-stone in economic and decision models (see e.g. [27,51]) dealing with subjective measurements of attitudes and perceptions. This semantic theory (initially stated in [37]), considers the polarity of concepts and the neutrality that holds in between, such that any concept is understood in relation to its positive and negative poles. The relation among opposite poles, examined under the term *bipolarity* [14,15], basically entails either a reciprocity or non-reciprocity among poles, where the former is commonly associated to a *bipolar univariate* model, and the latter to a *unipolar bivariate* one (see e.g. [26]).

In a general sense, bipolarity refers to the meaning of concepts and the nature of positive and negative knowledge. Hence, it is deeply related with that which we consider to be valid or even relevant knowledge. Take for example the intuitionistic philosophical–mathematical position (referring to a position defended in the last 19th and early 20th century by H. Poicaré and L.E.J. Brouwer), where the truth/provability of *P* can only be associated to the explicit mathematical construction of *P*. In consequence, the proof of the impossibility of *P* cannot be taken as the proof of its negative affirmation not-*P* [5], unless a certain reciprocity between *P* and not-*P* is *assumed* (this led to a philosophical discussion on the validity of the principle of the excluded middle, where it holds either *P* or not-*P*, and of some mathematical demonstrations that make use of it, like e.g. the existence of non-denumerable sets).

Now consider the meaning of gains and losses and how these concepts can be used in natural language. Consider a situation where we have the amount of 100c, and make a bet where we lose 20c and end up with 80c. Then it can be said that "we have losses of 20c". Now assume that we make a bet planning to win 100c more, but fail to win, although we manage to keep the initial 100c we started with. Then, if we understand that everything that is not gains automatically becomes losses, it could be said that"we have losses of 100c", even though we keep the same 100c that we started with. But if we distinguish between gains, not-gains, losses and not-losses, then, for the second case, it can be said (with proper sense) that "we have not gained 100c" and that "we have no losses".

As it has been recently stated in decision theory and mathematical literature (see e.g. [14,26]), the bipolar univariate model makes use of a one-dimensional scale whose end-points are opposite references. These references are taken to be reciprocal, such that the known value of one of them entails (by complementation) the value of the other. Then, in the univariate model the measurement of the meaning of a concept can be negative, neutral (neither

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negative nor positive) or positive. In consequence, its meaning cannot be positive and negative at the same time. On the other hand, the unipolar bivariate model allows a more general measurement, based on two unipolar scales respectively measuring positive and negative aspects, such that the concept can have a positive, negative, neither positive nor negative, or both positive and negative meanings at the same time.

At the current state of things, bipolarity has been examined following a typology where bipolarity types I, II and III are proposed [14,15]. The first two respectively refer to the above mentioned univariate and bivariate models, while the third one stands as a proposal on its own (it still remains to be further explored, as suggested in [35,39]). At first instance, type III bipolarity seems to refer to a bivariate model with more than one pair of opposite references, possibly describing the different and *non-reciprocal* sources of information by the multiple dimensions building up the meaning of concepts. As it seems, this third type is the most general setting for modeling complex concepts, due to the fact that the meaning of concepts emerges from the multiple positive and aversive stimuli composing perceptions and emotions [4,8,45]. Then, the different neutral states holding in between the opposite poles play a determinant role for understanding the respective concepts, configuring a pertinent valuation structure for explaining the available data.

From a general perspective, building the valuation structure for measuring the meaning of concepts entails that negative information/evidence exists *independently* from the positive one (i.e., from the intuitionistic point of view, the construction of a proof on the impossibility of P does not imply having a proof for the affirmation of not-P). Hence, as a consequence of such independence, an unavoidable step in order to understand concepts and reality is to modelize the neutral states that arise in between such opposites, not necessarily assuming a reciprocal relation between the positive and the negative information/evidence.

For example, the meaning of a reference concept C can be examined by its decomposition into the pairs Q and V, but also into pairs A and Z (and notice that any pole/concept, such as Q and V, is at the same time susceptible of being further decomposed, e.g. into Q^+, Q^- and V^+, V^-, respectively). Here, let C = "preference", such that C can be understood regarding its decomposition into the opposite poles/concepts of Q = "desire" and V = "non-desire", but also regarding the poles/concepts of A = "desire", and Z = "rejection", or even W = "want" and N = "need" (this example will be developed along the paper, extending the initial proposal of [21]). Notice that under the univariate model (bipolarity type I), the verification of C can only occur with respect to a given pair such as Q and V, where it can never hold that C is both Q and V. Then, if the *ambivalence* between Q and V should be represented, where C can be both Q and V, an independent *non-reciprocal* measurement of both Q and V would be required, and the same happens for A and Z, and for any other pair of *meaningful opposites* (a term firstly introduced in [37]), where such complexity builds on as much as it is required.

Hence, given a pair of opposite references and their associated poles representing *antagonistic* [40] perceptions of reality, the meaning of concepts is represented based on the same cognitive-emotional process of the brain and its treatment of pleasant and aversive stimuli. In fact, it has been observed by neurologists that such stimuli are processed separately in different physical areas of the brain (see e.g. [36,52]), as two independent perceptions, even combining both types of stimuli through distinct neural networks causing reactions and emotions leading to specific behaviors [2,10,30,53]. In this sense, different pleasant and unpleasant affective components of the same sensory stimulus may provide the inputs of human decision making [25]. In decision theory, some well-known examples referring to bipolar knowledge are Prospect Theory [28,51], the Choquet integral with respect to bi-capacities [27], and Partial Comparability Theory [42,48–50].

In this paper we examine *paired structures* [32] for the representation of bipolar knowledge and its role for subjective decision making. Paired structures allow representing and measuring the meaning of concepts, without imposing linearity among their associated opposite poles as it occurs in the (univariate/type-I) bipolar model [14,26,37]. Therefore, by means of paired structures, the verification/measurement of the meaning of a concept can be properly computed, where the semantics of the valuation structure is well specified regarding the relation among opposites together with their characteristic *neutrality*. We stress the point that neutrality in our sense should not be confused with the neutral element of bipolar valuation scales [14,26,37], as the neutral category we refer to characterizes the semantic relation between poles.

In order to specify the semantic structure of bipolar models and learn the meaning of concepts, we present in Section 2 the basic and necessary notions regarding the proposal of paired structures. Then in Section 3 we model preferences with paired structures, examining in Section 4 extended preference structures and their characterization of conflicting situations. Then we recapitulate by exploring a numerical example in Section 5 for making sense of data and learning preferences for decision making. Finally we give some final comments for future research.

2. Paired structures and the meaning of concepts

Bipolar scales (in the traditional sense of [14,26,37]) have been used to measure the meaning of concepts. Here we examine both the representation and the measurement of the meaning of concepts under the building process of paired structures. From this standpoint, a representation process refers to the meaning/use of stimuli correlating with perceptions and concepts, while a measurement process refers to the meaning/use of concepts for making observations and verifying their occurrence.

Based on this representation of meaning, opposite *paired* concepts (Q, V) *emerge* [6] from an initial *ignorance* category Ω, which is taken here as the point of departure of any learning process [33]. This is the basic structure supporting the paired structures, allowing making sense of data according to the different neutral situations emerging jointly with the opposite poles and their particular semantic relation (see Fig. 1).

Then, observations can be collected and measured with proper sense, being valued according to the previous specification of the semantical structure and the meaning of the concept giving sense to them (the observations or the objects of interest). This enables

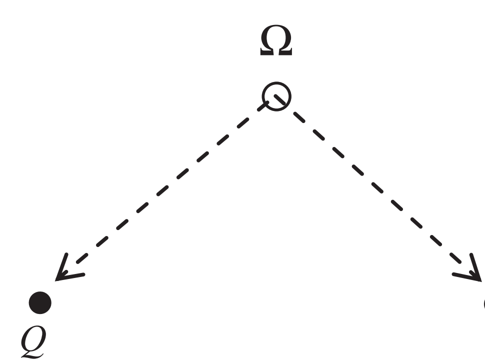

Fig. 1. Paired structure built from the ignorance concept Ω and opposite poles Q and V.