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An intelligent interactive approach to group aggregation of subjective probabilities

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ABSTRACT

We are concerned with the problem of obtaining a consensus subjective probability distribution from the individual opinions of a group of agents about the subjective probability distribution. We provide an iterative interactive algorithm that allows the agents to come to consensus formulation for the subjective probability distribution.

1. Introduction

We are concerned here with a problem in group decision making where we have a number of individual agents each of who have their own opinion about the subjective probability distribution with respect to the occurrence of an outcome from a finite set X possible outcomes. Our objective is to interactively combine the opinions of these individual agents about this subjective probability distribution to obtain a consensus subjective probability distributions. A number of researchers have looked at this problem [1–7]. In [2] Clemen and Winkler provide a very comprehensive and thoughtful analysis of this problem in which they discuss a number of approaches to providing this group subjective probability distribution. One approach is a formal mathematical type method based on an aggregation of the individual agents probability distributions. The second general type are behavioral or group decision making approaches. Typical of these are methods such as the Delphi method [8–11]. These involve interaction between the experts to come to agreement on the probability distribution. Here one may have a kind of moderator whose job is to encourage the participating agents to come to some agreement. This approach generally involves an iterative method where the agents, guided by the moderator, are encouraged to change their inputs to try to concur on a probability distribution. Here we present an approach in the spirit of the interactive Delphi method [8–11] but one having a more formal mechanism. Thus here we use iterative rounds where the agents can provide modified versions of their subjective probability distribution that allows them to take into account the current group aggregated probability distribution. Instead of having a moderator our approach has a formal mechanism that encourages a convergence of the individual agents subjective distributions to a consensus. This mechanism rewards those agents whose revised subjective probability distributions are the most compatible with the current group aggregated probability distribution from the previous iteration. It is very suitable for the types of automated negotiations discussed in [12].

We note that our ideas presented here have particularly benefited from our earlier work on multi-agent negotiations [13] and our prior work on group decision making with Pasi [14].

While our work is specifically focused on the problem of obtaining a group aggregation of subjective probability distributions we point out some very interesting work by Chiclana and his collaborators [15–22] on the problem of obtaining a group consensus of preferences in the context of decision making in which some closely related ideas have been investigated.

We want to make one technical point. We are actually combining the "opinions" of the individual agents with respect to the subjective probability distribution. As we shall see these opinions are manifested in each round as a particular subjective probability distribution. These opinions can be viewed as a kind of imprecise

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belief about the probabilities, a little bit in the spirit to second order probabilities. The fact that the underlying beliefs are opinions, imprecise, is what makes it possible for the agents to modify their provided subjective probability distribution on each round. Thus while formally we shall be working with precise subjective probability distributions to obtain a precise consensus subjective probability distributions underlying the process are these imprecise agent opinions about the probabilities.

The paper is organized as follows. We first provide a vector view of a probability distribution which we shall find useful throughout the paper. We introduce some measures of compatibility between probability distributions. We next our basic protocol for negotiations. We next describe our procedure for aggregation of the agent-supplied probability distributions. We next discuss various stopping rules. We finally discuss various alternative ways for aggregating the agent-supplied probability distributions.

2. Vector representation of a probability distribution

Assume P is a probability distribution on the space $X = \{x_1, \ldots, x_n\}$. Here then for each x_j, p_j is the probability of x_j where each $p_j \in [0, 1]$ and $\sum_{j=1}^{n} p_j = 1$. For our purposes in the following we shall find it convenient to present the probability distribution as an n dimensional vector, $P = [p_1, \ldots, p_n]$. Here this vector has the special properties that all the components are values in the unit interval and their sum is one.

We now recall some basic operations on these vectors. If $Q = [q_1, \ldots, q_n]$ is another probability distribution on the same space X then the weighted average of these two probability distributions $R = wP + (1 - w)Q$, with $w \in [0, 1]$, is another probability distribution on X where $r_j = wp_j + (1 - w)q_j$ is the probability of x_j.

Another operation on vectors is the dot product. If P and Q are two vectors then

$$P \bullet Q = \sum_{j=1}^{n} p_j q_j.$$

Here we see the dot product is a scalar value. A special example of the dot product is the case where we have $P = Q$, in this case $P \bullet P = \sum_{j=1}^{n} p_j^2$.

An important concept that is obtained from the dot product is the idea of a norm of a vector. The norm of the vector P is denoted as $\|P\|$ and defined as $\|P\| = \sqrt{P \bullet P} = \sqrt{\sum_{j=1}^{n} p_j^2}$. The norm is referred to as the Euclidean length of the vector. Because of the special properties of the probability distribution vector, $p_j \in [0, 1]$ and $\sum_j p_j = 1$, it can be shown that the maximal value of $\|P\|$ occurs when there is one $p_j = 1$ and all other $p_i = 0$. In this case $\|P\| = 1$. In addition the minimal value of $\|P\|$ occurs when all $p_j = 1/n$ and this has $\|P\| = \sqrt{\sum_{j=1}^{n} \left(\frac{1}{n}\right)^2} = \sqrt{\frac{1}{n}}$. It is interesting to note that the maximal value of $\|P\|$, is independent of the dimension of the vector, while the minimal value depends on the dimension. The larger the dimension of the probability vector, the smaller its minimal value.

An intuitive understanding of the situation can be had if we look at the two-dimensional probability vector as shown in Fig. 1. Because of the nature of probability vector, $p_1 + p_2 = 1$ the only allowed vectors emanating from the origin must terminate on the line $p_1 + p_2 = 1$. We see as the vector moves to the extremes of this line, $p_1 = 1$ or $p_2 = 1$ the value of its norm increases while as it moves to the midpoint of the line $p_1 = p_2$ the norm decreases.

If P and Q are two probability vectors it is known that

$$\mathrm{Cos}(\theta) = \frac{P \bullet Q}{\|P\| \, \|Q\|}.$$

Here θ is the angle between the vectors P and Q [23]. In Fig. 2 we illustrate this for the two dimensional case. Here we see that for these probability vectors $\theta \in [0, \pi/2]$. For this range of θ it is well known that $\mathrm{Cos}(\theta) \in [0, 1]$. We see that if $P = Q$ then $P \bullet Q = \|P\|^2$ and $\mathrm{Cos}(\theta) = 1$. It is well known that in this case $\theta = 0$. If P and Q are orthogonal, $P \bullet Q = \sum_j q_j p_j = 0$, then $\frac{PQ}{\|P\|\|Q\|} = \mathrm{Cos}(\theta) = 0$ in which case $\theta = \pi/2$. We observe that if P and Q are orthogonal then for any x_j such that $p_j \neq 0$ we have $q_j = 0$ and similarly for any x_j such that $q_j \neq 0$ we have $p_j = 0$. We observe if P is such that $p_j = 1$ for x_j and Q is such $q_k = 1$ for $x_k, x \neq j$ then P and Q are orthogonal.

In the case of this probability vectors it is interesting to note that the norm is related to a measure of entropy known as the Gini entropy [24], defined as

$$G(P) = 1 - \sum_{j=1}^{1} p_j^2 = 1 - \|P\|^2.$$

Here we see the larger $\|P\|$, the less the uncertainty, the more the information associated with the distribution. If P and Q are two probability vectors the Gini cross entropy is defined as $G(P, Q) = 1 - P \bullet Q$.

3. On multiple subjective probability distributions

A task that can arise in group decision-making environments is the combining of multiple subjective probability distributions provided by members of the group to obtain a group subjective probability distribution. Thus here if we have a set $X = \{x_1, x_2, \ldots, x_n\}$ of possible outcomes and a group of agents, $E_1, \ldots, E_q$, where each agent provides a subjective probability distribution over the space X. Here if P_i is the probability distribution provided by agent E_i then $P_i = [p_{i1}, p_{i2}, \ldots, p_{in}]$ where p_{ij} is the subjective probability that agent i assigns to outcome x_j. As with any probability distribution the components of each P_i satisfies, $p_{ij} \in [0, 1]$ and $\sum_{j=1}^{n} p_{ij} = 1$.

In [2] Clemen and Winkler provide a very comprehensive and thoughtful analysis of this problem. These authors discuss a number of approaches to providing this group subjective probability distribution. One approach is a formal mathematical type method that is based on an aggregation of the individual agents probability distributions satisfying certain required properties. The second type approach is what the authors in [2] referred to as a behavioral or group decision making approach. Typical of this approach are

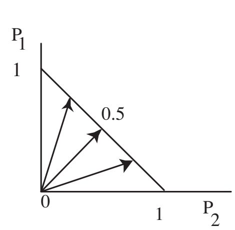

Fig. 1. Typical probability vectors.

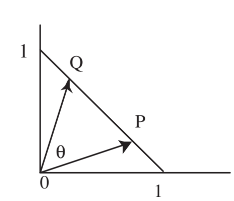

Fig. 2. Angle between two probability vectors.