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A two-grade approach to ranking interval data

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ABSTRACT

Ranking decision for interval data is a very important issue in decision making analysis. In recent years, several ranking approaches based on dominance relations have been developed. In these approaches, a dominance degree and an entire dominance degree are employed. However, one cannot obtain the complete rank of objects. To address this problem, this work will propose a two-grade approach to ranking interval data. In this approach, we keep the ranking result induced by the entire dominance degree in the first grade, and then refine the objects that cannot be ranked through introducing a so-called entire directional distance index. An example and a real case are employed to verify the effectivity of the two-grade ranking approach proposed in this paper.

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1. Introduction

In reality, one often encounters a number of alternatives which need to be evaluated on the basis of a set of criteria in investment decision [28,31,39,41], universities ranking [10,34], road safety risk evaluation [9], and so on. In these cases, the alternatives and the related criteria are often combined to a data table. In decision making, one needs to rank these alternatives through using some criteria that are characterized by attributes in the data table according to an increasing or a decreasing preference. This kind of decision making tasks are called ranking decision, which is becoming an important research point in decision making analysis. At present, ranking decision has been widely used in economy, management, engineering and other broad areas.

For effective and rational ranking decision, many decision making methods have been developed, which include TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [13,51], AHP (Analytic Hierarchy Process) [16,32,35], ELECTRE (ELimination Et Choix Traduisant la REalité) [33,37], and the methods based on fuzzy set theory [3,10–12,52,39], etc.

In the past twenty years, rough set theory introduced by Pawlak [24–27], has increasingly played an important role in the field of decision making analysis. One of its prominent advantages is to effectively deal with vagueness and uncertainty information without requiring any prior knowledge. As we know, Pawlak's rough set theory does not consider attributes with preference-ordered domains, that is, criteria. To solve the problem of ranking decision, several extended rough set models have been developed in the literature. Greco et al. [5–8] proposed an extension of rough set theory induced by a dominance relation, called a dominance-based rough set approach (DRSA), in which the ordering properties of criteria are taken into account. In what follows, we briefly review several works related to dominance-based decision making. By adding order relations on attribute values, Yao et al. [34,50] studied ordered information tables, and raised a convenient model to mine ordering rules through transforming an ordered information table into a binary information table. Yang et al. [47] introduced a similarity dominance relation, and developed a new dominance-based rough set model in incomplete ordered information systems. Hu et al. [11,12] presented a fuzzy preference rough set model by integrating fuzzy preference relations with a fuzzy rough set model. Moreover, evaluation on decision performance is also an important task in rough set theory [30,40,42,44]. In Ref. [44], concepts of knowledge granulation, knowledge entropy and knowledge uncertainty have been given to measure the discernibility ability of different knowledge in ordered information systems. Under the condition of homomorphism, Wang et al. [40] researched data compression in ordered information systems to perform equivalent attribute reductions and rule extraction in the smaller compressed image database for improving efficiency and saving decision-making

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costs. In a word, the dominance-based rough set theory has contributed a basic theoretical framework for ranking decision.

In decision making analysis, we often need to deal with various types of data sets, in which objects may be characterized by single value, set value, null value, or interval value [2,14,15,17,18,28–31,36,45–49]. Among these kinds of data sets, interval data is an important class of data and a generalized form of single-valued data. Hence, how to rank objects with interval values is a very desirable issue.

As mentioned above, although many results have been developed in the context of interval ordered information systems, how to rank objects using a dominance relation has not been reported. To address this problem, Qian et al. [28] proposed a ranking approach for all objects based on dominance classes and the entire dominance degree. This is the first attempt to rank objects with interval values. This method adopts a cautious decision strategy, in which we say that one object is superior to the other object if and only if the value of one object is dominant than that of the other under each attribute. Obviously, this approach is credible because it meets with practical decision situations, in which the risk aversion is one of major characteristics for decision makers.

However, it can be seen, from Qian's work [28], that the final rank obtained is not a complete rank, in which there may exist several objects being put into the same place. To overcome this drawback, we will further develop a new version of Qian's ranking approach according to the following two motivations.

(1) In practical issues, decision makers often want to get a complete rank of objects according to a user's requirement. A complete rank of objects will be helpful for obtaining a more satisfactory decision scheme. This opinion can be illustrated by using ranking decision of investment projects [31]. Generally, a complete rank of investment projects is necessary since decision makers only have limited capital. If several investment projects lie in the same place in final ranking result, that will confuse decision makers. Therefore, how to obtain a complete rank needs to be further addressed in the context of interval ordered information systems.
(2) In the process of looking for a complete rank, the rank induced by the entire dominance degree should be remained. As we know, a cautious ranking project is often desirable for the vast majority of decision makers. Hence, we argue that looking for a complete rank should be based on the cautious rank. For this reason, we will establish a two-grade ranking approach considering the property of rank preservation.

Therefore, in this paper, our objective is to develop an approach to obtaining a complete rank of objects with interval values. In this study, we first introduce the concept of a directional distance index and give some of its nice properties. Then, we define an ordered mutual information to calculate the weight of each criterion and the directional distance index with weights. Based on this consideration, we propose a two-grade approach to ranking objects with interval values. Finally, we also employ a real case about stock selection for verifying the effectivity of the proposed approach in this paper.

The remainder of this paper is organized as follows. Section 2 reviews some preliminary concepts and important properties of interval ordered information systems. Section 3 establishes a two-grade approach to ranking completely objects with interval values by combining a directional distance index with a dominance degree. In Section 4, through introducing weights of criteria based on an ordered mutual information, we propose a more rational two-grade approach to ranking completely interval data. In Section 5, we use a stock selection case to illustrate how to make a decision by using the ranking approach proposed in this paper. Finally, Section 6 concludes this paper with a remark.

2. Preliminaries

In this section, we briefly review some basic concepts and important properties of interval ordered information systems.

An *information system* (IS) is a quadruple $S = (U,AT,V,f)$, where U is a finite non-empty set of objects and AT is a finite non-empty set of attributes, $V=\bigcup_{a\in AT}V_a$ and V_a is a domain of attribute a, $f: U\times AT\to V$ is a total function such that $f(x,a)\in V_a$ for every $a\in AT$, $x\in U$, called an information function [31]. An information system is called an *interval information system* (IIS) if V_a is a set of interval numbers. We denote $f(x,a)\in V_a$ by

$$f(x,a)=[a^L(x),a^U(x)]=\{p|a^L(x)\leqslant p\leqslant a^U(x),a^L(x),a^U(x)\in\mathbf{R}\}.$$

It is the interval number of x under the attribute a.

Here, single-valued information systems, in which $f(x,a)=a^L(x)=a^U(x)$, can be seen as a special form of interval information systems. Example 2.1 shows an interval information system.

Example 2.1 [28]. An interval information system is listed in Table 1, where $U=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10}\}$, $AT=\{a_1,a_2,a_3,a_4,a_5\}$.

Definition 2.1. An interval information system is called an *interval ordered information system* (IOIS) if all attributes are criteria, that is, the domain of each attribute is ordered according to an increasing or a decreasing preference.

It is assumed that the domain of a criterion $a\in AT$ is completely pre-ordered by an outranking relation $\succcurlyeq_a$; $y\succcurlyeq_a x$ means that y is at least as good as (outranks) x with respect to the criterion a. Furthermore, we define $y\succcurlyeq_A x\Longleftrightarrow\forall a\in A(A\subseteq AT),y\succcurlyeq_a x$.

Based on the above illustration, we review the dominance relation that identifies dominance classes especially in an interval ordered information system. In a given IOIS, we say that y dominates x with respect to $A\subseteq AT$ if $y\succcurlyeq_A x$, and denoted by $yR_A^{\geqslant}x$. That is

$$\begin{aligned}R_A^{\geqslant}&=\{(y,x)\in U\times U|a_1^L(y)\geqslant a_1^L(x),\ a_1^U(y)\geqslant a_1^U(x)\\&\quad(\forall a_1\in A_1);\ a_2^L(y)\leqslant a_2^L(x),\ a_2^U(y)\leqslant a_2^U(x)\ (\forall a_2\in A_2)\}\\&=\{(y,x)\in U\times U|(y,x)\in R_A^{\geqslant}\},\end{aligned}$$

where the attributes set A_1 according to increasing preference and A_2 according to decreasing preference, and $A=A_1\cup A_2$.

According to the definition of $R_A^{\geqslant}$, the dominance class $[x]_A^{\geqslant}$ which is the set of objects dominating x can be induced as follows

$$\begin{aligned}[x]_A^{\geqslant}&=\{y\in U|a_1^L(y)\geqslant a_1^L(x),\ a_1^U(y)\geqslant a_1^U(x)(\forall a_1\in A_1);a_2^L(y)\\&\quad\leqslant a_2^L(x),\ a_2^U(y)\leqslant a_2^U(x)(\forall a_2\in A_2)\}=\{y\in U|(y,x)\in R_A^{\geqslant}\}.\end{aligned}$$

Analogously, $R_A^{\leqslant}$ and $[x]_A^{\leqslant}$ can be defined too.

From the definitions of $R_A^{\geqslant}$ and $[x]_A^{\geqslant}$, a partial order can be defined on the attribute set. Let $S=(U,AT,V,f)$ be an IOIS and A,

Table 1
An interval information system.

U	a_1	a_2	a_3	a_4	a_5
x_1	1	[0,1]	2	1	[1,2]
x_2	[0,1]	0	[1,2]	0	1
x_3	[0,1]	0	[1,2]	1	1
x_4	0	0	1	0	1
x_5	2	[1,2]	3	[1,2]	[2,3]
x_6	[0,2]	[1,2]	[1,3]	[1,2]	[2,3]
x_7	1	1	2	1	2
x_8	[1,2]	[1,2]	[2,3]	2	[2,3]
x_9	[1,2]	2	[2,3]	[0,2]	3
x_{10}	2	2	3	[0,1]	3

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