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Mathematical programming methods for consistency and consensus in group decision making with intuitionistic fuzzy preference relations

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ABSTRACT

In group decision making (GDM) with intuitionistic fuzzy preference relations (IFPRs), the consistency and consensus are two key issues. This paper develops a novel method for checking and improving the consistency of individual IFPRs and the consensus among experts. To measure the consistency degree of IFPRs, a consistency index is introduced and then an acceptable consistency is defined. For an IFPR with unacceptable consistency, a mathematical programming approach is developed to improve its consistency. To evaluate the consensus degree among experts, a consensus measure is presented by the proximity degree between one expert and other experts. When several individual IFPRs are unacceptable consistent or consensus is unacceptable, a goal program is built to improve the consistency and consensus simultaneously. By the consistency and proximity degrees of individual IFPRs, experts' objective weights are determined. Combining the experts' subjective weights, the experts' comprehensive weights are derived. Then, an intuitionistic fuzzy geometric weighted mean (IFGWM) operator is proposed to integrate individual IFPRs into a collective one. Moreover, an attractive property is proved that the collective IFPR is acceptable consistent if all individual IFPRs are acceptable consistent. Two examples are provided to illustrate the validity of the proposed method.

1. Introduction

Group decision making (GDM), where several experts are involved to select the best alternative(s) from a set of finite feasible alternatives, is widely used to collect the wisdom of more people in modern enterprise management. In GDM, once the set of feasible alternatives is identified, experts are called to express their opinions or preferences on such a set. For expressing preferences, in literature [28], Ureña et al. pointed that Millet [22] conducted a comparison study between different alternative preference elicitation methods and concluded that pairwise comparison methods are more accurate than non-pairwise methods. By pairwise comparisons between alternatives, a preference relation is built. The classical preference relations are multiplicative preference relation (MPR) [9,24,37] and fuzzy preference relation (FPR) [7,8,13,21,27], whose pairwise comparisons are single numeric values. However, due to the uncertainty of objects and the vagueness inherent in human thinking, it is more flexible to express the pairwise comparisons as a certain information granule (say, interval, fuzzy set, rough set and alike) [7]. Thus, interval-valued preference relation (IVPR) [2,15,32], triangular fuzzy complementary preference relations (TFCPRs) [35] and linguistic preference relations (LPRs) [20,23] emerged.

Although these preference relations have many advantages, each element of them only utilizes a membership degree to describe the degree of one alternative preferred over the other. As a result, the experts' hesitations or indeterminacies are often neglected. To circumvent this issue, Szmidt and Kacprzyk [25–26] presented IFPR whose elements are intuitionistic fuzzy values (IFVs) and can express the membership, non-membership and hesitant degrees jointly. Therefore, IFPR is more natural and flexible to describe the uncertainties of pairwise comparisons between alternatives in decision making. In recent years, GDM with IFPRs has received much attention and becomes a hot research topic. Generally, to solve a GDM problem with IFPRs, it is needed to perform three processes [16].

(1) The consistency checking and reaching process of individual IFPRs. The aim of this process is to guarantee the transitivity and rationality of the experts' preferences

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which could avoid misleading priority weights of alternatives. For checking the consistency of IFPRs, it is necessary to define the consistency. From existing research achievements, there are two main types of consistency: multiplicative consistency and additive consistency. The ways of defining the consistency can be roughly divided into two categories: one is based on the relations between the elements of IFPRs and the priority weights [11,12,38]. The other is based on the transitivity of preference relations. For example, Xu et al. [39] proposed a multiplicative consistency definition by extending the multiplicative transitivity consistency of FPRs [27]. Liao and Xu [17] improved the consistency definition proposed in [39]. Wu and Chiclana [36] proposed a new multiplicative consistency of IVPRs and then defined another multiplicative consistency of IFPRs based on the transformation between IVPRs and IFPRs. Motivated by the additive consistency of FPRs, Wang [31] defined an additive transitivity consistency of IFPRs.

Although several consistency definitions of IFPRs were introduced, the research on checking and reaching the consistency of IFPRs is still relatively little when IFPRs are inconsistent. Up to date, Wu and Chiclana [36] and Liao et al. [16] respectively presented two different consistency indices of IFPRs for checking the consistency. Meanwhile, Liao et al. [16] designed an iterative algorithm for reaching the consistency.

(2) The consensus checking and reaching process of a group. The consensus process is a preferable process to eliminate the non-consensus among experts, find and solve potential problems in a group decision-making process [43]. Chiclana et al. [10] and Mata et al. [20] argued that the consensus has two meanings: one is defined as the full and unanimous agreement of all the experts regarding all the feasible alternatives; the other is referred to the judgments arrived at by 'most of' those concerned. Cabrerizo et al. [6] divided the consensus into hard consensus and soft consensus, which are corresponding to the above two meanings, respectively. Hard consensus is viewed as a utopia since it adopts 0 (no consensus) or 1 (full consensus) to measure consensus. In contrast, soft consensus is more pragmatic because it assesses the consensus degree in a more flexible way. According to the soft consensus, different consensus methods were proposed in diverse fuzzy scenarios, such as FPRs [7], TFCPRs [35] and LPRs [6]. Recently, Herrera-Viedma et al. [14] reviewed most soft consensus methods in fuzzy environment. Although many consensus methods have been presented for solving consensus problems under fuzzy environment, Cabrerizo et al. [5] pointed that challenges still exist, for example, consensus in social network. Although Cabrerizo et al. [5] did not discuss the challenge consensus in IFPRs faces, the challenge also exists.

For the consensus in social network, Wu et al. [34] constructed a trust based consensus model to build consensus in incomplete linguistic information context. For the consensus in IFPR environment, the existing research primarily focused on checking the consensus (agreement) level among experts [16,25,26,36], while the methods for improving the consensus level are very few. To the best of our knowledge, only Liao et al. [16] developed a consensus reaching procedure and Wu and Chiclana [36] designed a feedback mechanism to help experts modify their some pairwise comparisons to improve the consensus level.

(3) The selection process. The aim of this process is to derive the best alternatives from the preference relations provided by experts. This process involves two different steps [10]: (i) aggregation of individual preferences and (ii) exploitation of the collective preferences. In the first step, a popular method is to use aggregating operators, such as multiplicative consistency induced order weighted averaging (MC-IOWA) operator [36], consistency and confidence IOWA operator [29] and so on. Occasionally, some aggregating rules [23] were employed. The second step is to exploit the collective IFPR to obtain the priority weights of alternatives by which the alternatives are ranked. In this regard, mathematical programs were often constructed to derive the priority weights of alternatives [3,11,12,17,30,31,40]. Recently, Wu and Chiclana [36] utilized intuitionistic quantifier guided non-dominance degree to rank alternatives. Ureña et al. [29] used intuitionistic fuzzy quantifier guided dominance degree and intuitionistic fuzzy quantifier guided non dominance degree to rank alternatives.

Although previous works have made significant contributions to solve GDM problems with IFPRs, there are following limitations.

(1) While checking the consistency degree of a given IFPR, existing consistency measures [16,36] have to employ the other IFPR which needs to be constructed by the complex computation. Therefore, it is not convenient enough to check the consistency degrees of IFPRs.
(2) When a given IFPR is unacceptable consistent, only Liao et al. [16] presented one iterative algorithm to improve its consistency degree. Nevertheless, it may be needed to repeat this algorithm for many times to repair this unacceptable consistent IFPR until it reaches acceptable consistency. It is time-consuming and needs heavy workload. In addition, it is unknown how much the repaired IFPR preserves the preference information of the initial IFPR.
(3) As for the consensus process, although several consensus indices were developed, there is no study on improving the consistency degrees of several individual IFPRs and consensus among experts simultaneously. Thus, it may be hard to formulate a decision result which is not only reasonable but also can be accepted by a group if some initial individual IFPRs in a GDM problem are unacceptable consistency and the consensus is unacceptable.
(4) In selection process, the experts' weights play an important role in aggregating individual IFPRs into a collective one. Existing methods [3,11,12] assigned experts' weights in advance, which cannot reflect the quality of preference information provided by experts in this decision making.

To overcome these limitations, this paper proposes a novel method for GDM problems with IFPRs. For checking the consistency of IFPRs, a new consistency index is introduced and then an acceptable consistency is defined. When an IFPR is unacceptable consistent, a mathematical programming approach is proposed to obtain an acceptable consistent IFPR from the initial IFPR through minimizing the deviation between the initial IFPR and the obtained IFPR. Afterward, to evaluate the consensus degree among experts, a consensus measure is developed by the proximity degree between one expert and other experts. Moreover, when several individual IFPRs are unacceptable consistent and the consensus is unacceptable, a goal programming model is established to improve the consistencies of several individual IFPRs and consensus jointly. Using the consistency and proximity degrees of individual IFPRs, the experts' objective weights are determined. Combining the experts' subjective weights, which reflect the powers or knowledge of experts in decision making, the experts' comprehensive weights are derived. Finally, a new intuitionistic fuzzy geometric weighted mean (IFGWM) operator is introduced and applied to integrate the individual IFPRs into a collective one. Furthermore, an attractive