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Confidence-consistency driven group decision making approach with incomplete reciprocal intuitionistic preference relations

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ABSTRACT

Intuitionistic preference relations constitute a flexible and simple representation format of experts' preference on a set of alternative options, while at the same time allowing to accommodate degrees of hesitation inherent to all decision making processes. In comparison with fuzzy preference relations, the use of intuitionistic fuzzy preference relations in decision making is limited, which is mainly due to the computational complexity associated to using membership degree, non-membership degree and hesitation degree to model experts' subjective preferences. In this paper, the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations are proved to be mathematically isomorphic. This result can be exploited to use methodologies developed for fuzzy preference relations to the case of intuitionistic fuzzy preference relations and, ultimately, to overcome the computation complexity mentioned above and to extend the use of reciprocal intuitionistic fuzzy preference relations in decision making. In particular, in this paper, this isomorphic equivalence is used to address the presence of incomplete reciprocal intuitionistic fuzzy preference relations in decision making by developing a consistency driven estimation procedure via the corresponding equivalent incomplete asymmetric fuzzy preference relation procedure. Additionally, the hesitancy degree of the reciprocal intuitionistic fuzzy preference relation is used to introduce the concept of expert's confidence from which a group decision making procedure, based on a new aggregation operator that takes into account not only the experts' consistency but also their confidence degree towards the opinion provided, is proposed.

1. Introduction

Intuitionistic fuzzy preference relations are based on the concept of intuitionistic fuzzy set that Atanassov introduced in [3] as an extension of the concept of fuzzy set. Due to its flexibility in handling vagueness/uncertainty, intuitionistic fuzzy set theory [4] has been extensively used in many areas, such as virtual medical diagnosis [11], pattern recognition [26], clustering analysis [30] and decision making [23,27–29]. For example in [10], Fujita et al. propose to model the user cognitive behaviour on mental cloning-based software using intuitionistic fuzzy sets.

Much research has been carried out in decision making with preferences modelled using fuzzy relations in comparison to using intuitionistic fuzzy relations. This is mainly to the longer existence of the former representation format of preferences in comparison to the second one. However, an additional cause for the lesser use of intuitionistic fuzzy preference relations in decision making is the increase computational complexity associated to the use of membership degree, non-membership degree and hesitation degree to model experts' subjective preferences. Notice that intuitionistic fuzzy preference relations are usually assumed to be reciprocal (Section 2).

A first objective of this paper is to prove the mathematical equivalence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations. This result can thus be exploited to use methodologies developed for fuzzy preference relations to the case of intuitionistic fuzzy preference relations and, ultimately, to extend the use of reciprocal intuitionistic fuzzy preference relations in decision making and to overcome the computation complexity mentioned above. In other word, this result will allow to take advantage of

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mature and well defined methodologies developed for fuzzy preference relations while leveraging the flexibility of reciprocal intuitionistic fuzzy preference relations to model vagueness/uncertainty. Indeed, an issue that can be addressed using the mentioned equivalence is the presence of incomplete reciprocal intuitionistic fuzzy preference relations in decision making.

Incomplete information as a result from the incapability of experts to provide complete information about their preferences [14,7] may happen more frequently than expected due to different reasons such as: experts not having a precise or sufficient level of knowledge of part of the problem, lack of time, difficulty to distinguish up to which degree one preference is better than other, or conflicting between alternatives, among others. In the literature, different approaches to deal with missing or incomplete information have been extensively studied for the case of using fuzzy preference relations as the representation format of preferences [25]. Most of the existing approaches are based on the selection of an appropriate methodology to 'build' the matrix, and/or to assign importance values to experts based not on the amount of information provided but on how consistent the information provided is [1,8,14,18,28].

The case of incomplete intuitionistic fuzzy preference relations has been addressed in literature in [29,28], where the above mentioned methodology to estimate missing information driven by the consistency was adopted. The main difference between both approaches resides in the way consistency of reciprocal intuitionistic fuzzy preference relations was modelled. On the one hand, in [29] a straight forward transposition of the multiplicative consistency property for fuzzy preference relations was proposed for the case of reciprocal intuitionistic fuzzy preference relations, which has been later proved to be incorrect [28], and publicly acknowledged by the authors that proposed it [31]. On the other hand, in [28] the concept of multiplicative consistency for reciprocal intuitionistic fuzzy preference relations was derived by formally extending the multiplicative transitivity property for fuzzy preference relations via the use of both the *Extension Principle* and *Representation Theorem* [35]. In this contribution, though, a different approach to incomplete reciprocal intuitionistic fuzzy preference relations is presented based on the aforementioned equivalence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations. The main advantage of the approach put forward here is that the isomorphic relation between reciprocal intuitionistic fuzzy preference relations and asymmetric fuzzy preference relations makes superfluous both the extension principle and the representation theorem that were required in [28], as well as being less computationally complex because there is no need to split the reciprocal intuitionistic fuzzy preference relations into two reciprocal fuzzy preference relations but one single asymmetric fuzzy preference relation.

A second objective of this paper is to develop a fuse approach of the information provided by the experts taking into account the confidence level of each expert in his/her own opinion, which is intrinsically connected to the information he/she provides [12], and which in the case of reciprocal intuitionistic fuzzy preference relations is linked to the associated hesitancy function. Obviously, the more confident the expert feels about his/her opinion the more relevant the opinion can be considered, and thus more importance should be allocated to it. This can be achieved in the aggregation phase of a group decision making model by implementing an appropriate confidence and consistency based induced ordered weighted average to compute the collective preferences [6,14,28].

The rest of the paper is set out as follows: Section 2 presents the main mathematical frameworks for representing preferences of interest, while Section 3 deals with the concept of consistency of fuzzy preference relations as needed throughout the rest of the paper. Section 4 demonstrates the mathematical equivalence between the set of reciprocal intuitionistic fuzzy preference relations and the set of asymmetric fuzzy preference relations, which is used in Section 5 to present a methodology to estimate missing values of reciprocal intuitionistic fuzzy preference relations. The hesitancy function is proposed in a confidence-consistency driven group decision making approach with incomplete reciprocal intuitionistic fuzzy preference relations whose application is illustrated with an example in Section 6. Finally, Section 7 includes an analysis of the proposed group decision making model, including some future work and draws conclusions.

2. Preference relations in decision making

In any decision making problem, once the set of feasible alternatives (X) is identified, experts are called to express their opinions or preferences on such set. Different preference elicitation methods were compared in [19], concluding that pairwise comparison methods are more accurate than non-pairwise methods since it allows the expert to focus only in two alternatives at a time. A comparison of two alternatives of X by an expert can lead to the preference of one alternative to the other or to a state of indifference between them. Obviously, there is the possibility of an expert being unable to compare them. Two main mathematical models based on the concept of preference relation can be used in this context. In the first one, a preference relation is defined for each one of the above three possible preference states (preference, indifference, incomparability), which is known as a preference structure on the set of alternatives. The second one integrates the three possible preference states into a single preference relation. In this paper, we focus on the second one as per the following definition:

Definition 1 (*Preference Relation*). A preference relation P on a set X is a binary relation $\mu_P : X \times X \to D$, where D is the domain of representation of preference degrees provided by the decision maker.

A preference relation P may be conveniently represented by a matrix $P = (p_{ij})$ of dimension $card(X)$, with $p_{ij} = \mu_P(x_i, x_j)$ being interpreted as the degree or intensity of preference of alternative x_i over x_j. The elements of P can be of a numeric or linguistic nature, i.e., could represent numeric or linguistic preferences, respectively [20]. The main types of numeric preference relations used in decision making are: crisp preference relations, additive preference relations, multiplicative preference relations, interval-valued preference relations and intuitionistic preference relations. In this contribution we are going to focus on the reciprocal intuitionistic fuzzy preference relations and their equivalence to a subclass of asymmetric fuzzy preference relations.

2.1. Fuzzy set and fuzzy preference relation

Definition 2 (*Fuzzy Set*). Let U be a universal set defined in a specific problem, with a generic element denoted by x. A fuzzy set X in U is a set of ordered pairs:

$$X = \{(x, \mu_X(x)) | x \in U\}$$

where $\mu_X : U \to [0,1]$ is called the membership function of A and $\mu_X(x)$ represents the degree of membership of the element x in X.

Notice that the degree of non-membership of the element x in X is here defined as $\nu_X(x) = 1 - \mu_X(x)$. Thus, $\mu_X(x) + \nu_X(x) = 1$.