



# Multiple criteria ranking and choice with all compatible minimal cover sets of decision rules



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## ABSTRACT

We introduce a new multiple criteria ranking/choice method that applies Dominance-based Rough Set Approach (DRSA) and represents the Decision Maker's (DM's) preferences with decision rules. The DM provides a set of pairwise comparisons indicating whether an outranking (weak preference) relation should hold for some pairs of reference alternatives. This preference information is structured using the lower and upper approximations of outranking ( $S$ ) and non-outranking ( $S^c$ ) relations. Then, all minimal-cover (MC) sets of decision rules being compatible with this preference information are induced. Each of these sets is supported by some positive examples (pairs of reference alternatives from the lower approximation of a preference relation) and it does not cover any negative example (pair of alternatives from the upper approximation of an opposite preference relation). The recommendations obtained by all MC sets of rules are analyzed to describe pairwise outranking and non-outranking relations, using probabilistic indices (estimates of probabilities that one alternative outranks or does not outrank the other). Furthermore, given the preference relations obtained in result of application of each MC set of rules on a considered set of alternatives, we exploit them using some scoring procedures. From this, we derive the distribution of ranks attained by the alternatives. We also extend the basic approach in several ways. The practical usefulness of the method is demonstrated on a problem of ranking Polish cities according to their innovativeness.

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## 1. Introduction

In this paper, we consider choice and ranking problems where alternatives are evaluated on multiple, often conflicting, criteria. Choice problems are oriented toward selecting a subset of the most prevailing alternatives, whereas in ranking problems one aims at imposing a preference order on the set of alternatives. The only conclusion which can be derived from the analysis of performances of the considered alternatives on multiple criteria is the dominance relation among them. Leaving many alternatives incomparable, this relation prevents, however, their clear ranking or straightforward discrimination between alternatives that should be selected and neglected. Thus, to work out a recommendation, the Decision Maker (DM) needs to enrich the dominance relation by providing

some extra preference information which is subsequently transformed into a mathematical preference model. The application of the preference model induces a preference structure on the set of alternatives. The ranking or choice recommendation can be derived from its proper exploitation. In Multiple Criteria Decision Aiding (MCDA), there exist three basic ways of modeling preference information coming from the DM: value functions [35], relational systems [42], or “if ..., then ...” decision rules [19,20,47].

These models need to faithfully represent the elements of DM's value system. In this perspective, they can be tuned using either direct or indirect preference statements. Since the previous need a considerable cognitive effort on the DM's part, the recent decision aiding methods are designed so that to accommodate indirect or incomplete preference information. In the context of multiple criteria ranking and choice, such preference information is composed of some exemplary decisions concerning a small subset of reference alternatives. Although these judgments may have different forms (for a review, see [9]), the majority of methods employ pairwise comparisons.

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### 1.1. Robustness analysis in value- and outranking-based multiple criteria ranking and choice methods

With indirect and incomplete specification of preferences, there are typically several instances of the preference model (i.e., functions, relations, or sets of rules) that are consistent with the preference statements. While all such compatible instances reproduce the preference information provided by the DM for reference alternatives, the recommendation delivered for the non-reference ones may vary significantly from one instance to another. The potential diversity of the suggested recommendation motivated the development of a framework for robustness analysis. Dealing with the plurality of compatible preference model instances in the context of ranking and choice problems has been already considered for two out of three preference models used in MCDA: a value function and an outranking relation.

When analyzing the robustness of recommendation obtained with a value-based preference model, we may identify such pairs of alternatives that the comprehensive value of the first one is at least as good as that of the second one for all value functions, and strictly higher for some value functions [27,55]. A similar result in the setting of Robust Ordinal Regression (ROR) corresponds to the necessary and possible preference relations [10,25]. Moreover, we may take into account ranks attained by the alternatives by indicating either potentially optimal alternatives (i.e., these which can be ranked first by some compatible value function) [27,37] or, more generally, the whole range of ranks for each alternative by conducting extreme ranking analysis [31]. Finally, within the framework of Stochastic Ordinal Regression (SOR), one may estimate probabilities of both preference relations and attaining some rank, using the Monte Carlo simulation [34,36].

When it comes to outranking-based ranking and choice methods, the robustness concern has been raised by Dias and Clímaco [11], Greco et al. [16], and Kadziński et al. [31]. These approaches admit indirect and partial preference information concerning parameters of the model used in ELECTRE (see, e.g., [13]) or PROMETHEE (see, e.g., [7]). Having constructed a set of relational systems compatible with the DM's preferences, they verify the possibility and necessity of an outranking relation for each pair of alternatives by checking if it holds for, respectively, at least one or all admissible combinations of parameter values.

### 1.2. Rule induction algorithms for multiple criteria ranking and choice

The above review proves that robustness analysis has been widely used as a decision aiding tool within value- and outranking-based ranking and choice methods. Nevertheless, it has not received due attention in the context of decision rules. This model has been introduced to decision analysis several years ago, quickly gaining popularity because of its explanation potential and recommendation formulated in a natural language. The use of decision rules in MCDA is inseparably connected with Dominance-based Rough Set Approach (DRSA) [19,47] (for some recent advances or applications of DRSA, see, e.g., [2,3,8,28,39,38,40]). It structures the data so that sets of alternatives (in case of sorting problems) or sets of pairs of alternatives (in case of ranking and choice problems) are represented by the lower and upper approximations of decision classes or preference relations, respectively [41].

In adaptation of DRSA to multiple criteria ranking [17], the DM provides a set of pairwise comparisons indicating whether an outranking (weak preference) relation should hold ( $S$ ) or not ( $S^c$ ) for some reference alternatives. Decision rules which are induced from the approximations of comprehensive outranking ( $S$ ) and non-outranking ( $S^c$ ) relations concern pairs of alternatives. Their

application on the set of alternatives yields a specific preference relation in this set. This relation needs to be further exploited with some ranking method that arranges the alternatives in a preference order (see, e.g., [6,12,51,52]).

Many algorithms for induction of decision rules have been introduced in the context of multiple criteria ranking approached with DRSA. The vast majority of these algorithms generates a minimal-cover (MC) set of minimal decision rules [5,17,51,52]. In this way, pairs of alternatives from the lower or upper approximations of outranking and non-outranking relations are described with the set of most general, complete and non-redundant “if ..., then ...” statements. However, there are usually multiple sets of rules satisfying these properties, and the existing algorithms select a single one in an arbitrary pre-defined way. Obviously, the ranking or choice recommendation that can be obtained for any compatible set of rules can vary significantly.

### 1.3. Content and plan of the paper

The aim of this paper is to introduce an approach for multiple criteria ranking and choice with all MC sets of rules compatible with the DM's indirect and incomplete preference information. Analogously to An and Tong [1], Greco et al. [17,16], Szelag et al. [51,52], we expect the DM to provide a set of pairwise comparisons stating the truth ( $S$ ) or falsity ( $S^c$ ) of the outranking relation for some reference alternatives. Thus exhibited preference information is treated as deterministic, and structured using the lower and upper approximations of outranking and non-outranking relations. Then, all MC sets of decision rules being compatible with this preference information are induced from the structured information, such that rules suggesting  $S$  are induced with the hypothesis that the lower approximation of  $S$  provides positive examples and the upper approximation of  $S^c$  provides negative examples, and vice versa in case of inducing rules suggesting  $S^c$ . The compatibility of inferred rule sets with the exhibited DM's preference information is due to the fact that these sets cover all pairs of reference alternatives from the lower approximation of  $S$  and  $S^c$ , respectively. In this regard, analogously to other MCDA methods based on indirect preference information, the supplied pairwise comparisons constrain the flexibility of compatible preference model instances. In our case, a compatible instance of the preference model is a minimal set of minimal rules covering all pairs of reference alternatives compared by the DM and included in lower approximations of  $S$  and  $S^c$  – it is called MC set of rules.

The recommendations obtained by all MC sets of rules are analyzed to describe the stability of outranking and non-outranking by means of the necessary and the possible, as well as by the probabilistic indices (estimates of probabilities that one alternative outranks or does not outrank the other). To pass from the robustness analysis of the preference relations imposed on the set of alternatives to the recommended ranking/choice, we exploit them using different scoring procedures and derive the distribution of ranks attained by the alternatives. We also extend the basic approach in several ways.

The approach presented in this paper can be seen as a rule-based counterpart of Robust and Stochastic Ordinal Regression methods with value- [25,34] and outranking-based [16] preference models. In this regard, our main contribution is in the phase of construction of the DM's preference model. Precisely, we extend the existing rule-based methods for multiple criteria ranking and choice [5,17,14,51,52] to take into account all compatible MC set of rules, and not only one such set. Note, however, that when using value- or outranking-based preference model, either all compatible preference model instances are considered implicitly by taking into account a set of Linear Programming (LP) constraints or one

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