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Optimal discrete fitting aggregation approach with hesitant fuzzy information

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ABSTRACT

As a novel and generalized fuzzy set, hesitant fuzzy set have received increasing attention and have been a popular research topic recently. However, many crossover calculations are needed to aggregate hesitant fuzzy information for multiple calculations in its general operation, whereas much more of these calculations are needed in hesitant fuzzy group decision making. This paper proposes optimal discrete fitting aggregation and simplified optimal discrete fitting aggregation as two new aggregation approaches to resolve this issue. The paper also distinguishes their similarities and differences, proves some desired properties, and provides the selected methods and modeling steps. Moreover, the length extended technology and the deviation function are developed as the applied basis of the new approaches. Two practical cases are provided at the end of this paper to demonstrate the application of the proposed approaches.

1. Introduction

Fuzzy set [33] theory has been proposed and extended to many other forms to better understand the vagueness and uncertainty of the real information by presenting objective mathematical symbols. In particular, hesitant fuzzy set (HFS) and hesitant fuzzy element (HFE) have received increasing attention and have been a popular research topic recently because the HFS is a novel and generalized fuzzy set. Some significant research results are obtained based on HFSs and HFEs [22,27,28,7,5,3,4,38,17,15,16,18,11–14,32,20,21,41,19,2].

To aggregate the hesitant fuzzy information and make decisions, Xia and Xu [25] gave six basic operations, and further proposed the hesitant fuzzy averaging (HFA) operator, the hesitant fuzzy geometric (HFG) operator, the hesitant fuzzy ordered weighted averaging (HFOWA) operator, the hesitant fuzzy ordered weighted geometric (HFOWG) operator, and the hesitant fuzzy hybrid aggregation (HFHA) operator, etc. In addition, Xu and Xia [29] investigated the hesitant fuzzy entropy measures to aggregate the hesitant fuzzy information. Based on the ideas of the quasi-arithmetic means [8] and the induced approach [31], Xia et al. [26] studied some induced aggregation operators for HFSs. In addition, Wang et al. [10] provided the multi-criteria outranking approach for HFSs. Chen et al. [3,4] studied the correlation coefficients of HFSs and their applications to clustering analysis. Bedregal et al. [1] provided the aggregation functions for typical HFEs. Moreover, Wei [24] and Zhang [36] proposed the hesitant fuzzy prioritized (HFP) operator and the hesitant fuzzy power aggregation (HFPA) operator, respectively. Based on the induced opinion, Zhang et al. [37] proposed the induced generalized hesitant fuzzy ordered weighted averaging (IGHFOWA) operator and the induced generalized hesitant fuzzy ordered weighted geometric (IGHFOWG) operator. By introducing Bonferroni mean (BM), Zhu et al. [40] and Zhou [39] investigated the weighted hesitant fuzzy Bonferroni mean (WHFBM) and the hesitant fuzzy reducible weighted Bonferroni mean (HFRWBM) respectively. Recently, Liao and Xu [12] creatively derived the subtraction and division operations over HFSs, then, four elementary operations of HFEs have been constructed. Based on these operations, Liao et al. [15] investigated the multiplicative consistency of a hesitant fuzzy preference relation (HFPR), and proposed the hesitant fuzzy hybrid arithmetical averaging (HFHAA) operator, the hesitant fuzzy hybrid arithmetical geometric (HFHAG) operator, the quasi HFHAA operator and the quasi HFHAG operator [11].

As stated in the above paragraph, many hesitant fuzzy operators have been proposed to aggregate hesitant fuzzy information. However, there is a common limitation when applying these operators, that is, many crossover calculations are needed [11–14], whereas much more of these calculations are needed in hesitant fuzzy group

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decision making, which are caused by the basic hesitant fuzzy addition and multiplication. To address this issue, Liao et al. [17,15,16,18] proposed the order hesitant fuzzy addition and multiplication, which just belongs to an improved approach. For example, to aggregate two simple HFEs $\{0.1, 0.5, 0.9\}$ and $\{0.2, 0.3, 0.4\}$ by addition or multiplication, nine calculations (or three calculations by Liao et al.'s method) are needed. If there are four similar HFEs, then nearly 81 calculations (or nine calculations by Liao et al.'s method) are needed. Much more calculations are needed in hesitant fuzzy group decision making. The specific reasons are demonstrated in detail and are analyzed in Sections 4 and 5.

For the aforementioned reasons, this paper proposes two fitting aggregation technologies with the hesitant fuzzy information, i.e., the optimal discrete fitting aggregation (ODFA) and the simplified ODFA (S-ODFA), to address the limitation analyzed in the previous paragraph. These fitting technologies use nonlinear programming to fit different HFEs, and obtain an optimal discrete fitting hesitant fuzzy element (O-HFE) to present these HFEs. Thus, the objective of aggregating hesitant fuzzy information in one-time calculation is achieved, which is an obvious advantage of our approach over the general method and Liao et al.'s improved method [15]. Before that, this paper provides an improved length extended approach (ILEA) to meet the point-to-point demand in the ODFA and the S-ODFA, and provides five corresponding decision-making models.

To do so, this paper is organized as follows: The score functions, the aggregation operators, and the aggregation operator shortages are analyzed in Section 2. A length extended technology is developed and the comparison laws are provided in Section 3. Based on these discussions, the ODFA and the S-ODFA are proposed, the desired properties of these approaches are proven, and the selected methods and modeling steps are provided in Section 4. In Section 5, an example is given to demonstrate their application. The paper ends in Section 6 with conclusions.

2. Analysis of HFEs

2.1. HFEs and their comparison laws

Torra [22] pointed out that the difficulty of establishing the membership when defining the membership of an element is not caused by the existing margin of error or some possibility distribution on the values, but because we have a set of possible values. For such cases, Torra proposed the concept of HFS:

Definition 1 [23]. Let X be a fixed set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of $[0,1]$.

To be easily understood, Xia and Xu [25] expressed the HFS by a mathematical symbol:

$$E = \{< x,\ h_E(x) > | x \in X\} \tag{1}$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to the set E. For convenience, Xia and Xu [25] called $h = h_E(x)$ a hesitant fuzzy element (HFE), and H the set of all HFEs. In addition, the following score function and comparison laws were defined:

Definition 2 [25]. For a HFE h, $s(h) = \frac{1}{\#h}\sum_{i=1}^{N} h_i$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Moreover, Farhadinia [6] provided a series of score functions, which include the arithmetic-mean score function, $s_{AM}(h) = \sum_{i=1}^{N} h_i \Big/ N = s(h)$, the minimum score function, $s_{\min}(h) = \min\{h_i\}$, the maximum score function, $s_{\max}(h) = \max\{h_i\}$, and the bounded sum score function $s_{BS}(h) = \min\left\{1, \sum_{i=1}^{N} h_i\right\}$.

In the following, an illustrative example is provided to indicate that the previous score functions do not always rank correctly:

Example 1. Let $h_1 = \{0.1, 0.4, 0.5, 0.6, 0.9\}$, $h_2 = \{0.1, 0.5, 0.9\}$ and $h_3 = \{0.1, 0.9\}$ be three HFEs. Clearly, h_1, h_2 and h_3 are not exactly equal.

By applying Definition 1 and other four score functions, the following conclusions can be obtained:

$$s_{AM}(h_1) = s_{AM}(h_2) = s_{AM}(h_3) = 0.5,\ s_{\min}(h_1) = s_{\min}(h_2) = s_{\min}(h_3) = 0.1$$
$$s_{\max}(h_1) = s_{\max}(h_2) = s_{\max}(h_3) = 0.9,\ s_{BS}(h_1) = s_{BS}(h_2) = s_{BS}(h_3) = 1$$

Then, $h_1 = h_2 = h_3$, which is contradictory and produces some illogical results.

Thus, HFEs still could not be further distinguished when they have the same score values. Then, similar to the accuracy function of IFS [9,30] and the variance function of HFS [17,15,16,18], a new deviation function is introduced in the following definition to distinguish HFEs effectively:

Definition 3. Let $H = \{h_i\}$ $(i = 1, 2, \ldots, n)$ be a HFS, and $h_i = \cup_{\gamma_{ij} \in h_i}\left\{\gamma_{ij}\right\} = \left\{\gamma_{ij}\right\}_{j=1}^{l(h_i)}$ be HFEs, where $l(h_i)$ denotes the number of elements in h_i. If $L(H) = \max\{l(h_i)\}$, then a deviation function $d(h_i)$ of a HFE in H is defined by

$$d(h_i) = \frac{\sum_{\gamma_{ij} \in h_i} |\gamma_{ij} - s(h_i)|}{L(H)} \tag{2}$$

Then, $d(h) \in [0,1]$. According to the order relation between two IFVs, the score function $s(h)$ and the deviation function $d(h)$, the comparison laws between two HFEs h_n and h_m are provided as follows:

If $s(h_n) > s(h_m)$, then $h_n > h_m$; if $s(h_n) = s(h_m)$, then (1) if $d(h_n) < d(h_m)$, then $h_n > h_m$; (2) if $d(h_n) = d(h_m)$, then $h_n \sim h_m$; (3) if $d(h_n) > d(h_m)$, then $h_n < h_m$.

Compared with the similar functions in the existing literature such as [17,15,16,18,38,39], the new deviation function has two properties:

Property 1 (*Extended Invariance*). *Let $H = \{h_i\}(i = 1, 2, \ldots, n)$ be a HFS and $h_i = \cup_{\gamma_{ij} \in h_i}\left\{\gamma_{ij}\right\} = \left\{\gamma_{ij}\right\}_{j=1}^{l(h_i)}$ be HFEs, where $l(h_i)$ denotes the number of elements in h_i. Then, $d(h_i) = d(\hat{h}_i)$, where $\hat{h}$ is the extended value of h derived by using ILEA.*

Proof. The proof of Property 1 is similar to Property 3. □

Property 2 (*Scale Invariance*). *Let $H_1 = \{h_i\}(i = 1, 2, \ldots, n)$ and $H_2 = \{h_j\}(j = 1, 2, \ldots, m)$ be two HFSs. If $d_1(h_k) \geqslant d_1(h_t)$ $(1 \leqslant k, t \leqslant n)$, then*

$$d_2(h_k) \geqslant d_2(h_t) \quad \text{and} \quad \frac{d_1(h_k)}{d_1(h_t)} = \frac{d_2(h_k)}{d_2(h_t)}$$

where $d_1(\cdot)$ and $d_2(\cdot)$ are the deviation functions of H_1 and H_2 respectively.

Proof. Based on the definition of the deviation function $d(h)$, we have