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Relations between granular reduct and dominance reduct in formal contexts

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ABSTRACT

One of the key issues of knowledge discovery and data mining is knowledge reduction. Attribute reduction of formal contexts based on the granules and dominance relation are first reviewed in this paper. Relations between granular reduts and dominance reducts are investigated with the aim to establish a bridge between the two reduction approaches. We obtain meaningful results showing that granule-based and dominance-relation-based attribute reducts and attribute characteristics are identical. Utilizing dominance reducts and attribute characteristics, we can obtain all granular reducts and attribute characteristics by the proposed approach. In addition, we establish relations between dominance classes and irreducible elements, and present some judgment theorems with respect to the irreducible elements.

1. Introduction

The theory of formal concept analysis (FCA) proposed by Wille [13,54] has been formalized into an efficient methodology for data analysis and knowledge processing. FCA has become an important and appealing theoretical undertaking in recent years. It has been applied to various domains such as data mining, information retrieval, knowledge acquisition, software engineering, and database management systems [1–4,6,10,12,15,20,23–25,28,29,47,59,63,64].

Attribute reduction is an important issue in the discovery of knowledge in information systems. In terms of formal contexts, attribute reduction is the search for a minimal attribute subset that preserves required concepts and their hierarchical structure by deleting irrelevant attributes from the database. Rapid growth of interest in attribute reduction in FCA is evidenced in recent years [9,11,27,37–41,52,57,66].

The theory of rough set (RS), proposed by Pawlak [44], is an extension of the classical set theory. RS is a tool useful for dealing with imprecision, vagueness and uncertainty of information. The original rough set theory does not consider attributes with preference-ordered domains. However, in reality, one often faces the ordering problems of objects. For this reason, Greco et al. [16–18] proposed a dominance-based rough sets approach (DRSA) to take into account the ordering properties of attributes and gave a general frame work. Greco et al. [19] presented a general model of rough approximations based on ordinal properties of membership functions of fuzzy sets, in which the classical rough set theory can be considered as a particular case.

Attribute reduction is a major topic in rough set research. An attribute reduct is a minimum subset of attributes that preserves required property of a given information system obtained under the entire set of attributes by eliminating attributes that are not essential for the classification of objects or decision rules. Based on different binary relation and different requirements, a large variety of models and approaches to knowledge reduction have been proposed in the last two decades, see for examples [7,21,26,35,42,32–34,48,56,58,65].

Both FCA and RS are analyzed based on binary information tables. Both deal with the problems of knowledge discovery and knowledge representation. Therefore, the relations between FCA and RS is an interesting research direction. This paper focuses on attribute reduts of FCA and RS, and investigate the relations between FCA reduts and RS reducts in a formal context. The meaningful results have been obtained.

The paper is organized as follows. In the next section, the related work is recalled. In Section 3, we first review some basic notions of formal concept analysis. We then briefly recall the definitions of granular reduction and the corresponding reduction theory in Wu et al. [57] that serves as a basis for subsequent

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analysis. In Section 4, we introduce dominance relation into formal contexts and discuss the relations between dominance class and the irreducible element. Based on dominance relation, attribute reduction and attribute characteristics of formal contexts is proposed in Section 5. In Section 6, we discuss approaches to attribute reduction in consistent formal decision contexts. We demonstrate applications of the proposed method in Section 7. We then conclude the paper with a summary and directions for future research in Section 8.

2. Related work

Attribute reduction in FCA have been investigated in several works in the literatures. We recall, Gediga and Wille [13] first gave the notions of reducible attributes and reducible objects by reducing columns and rows in formal contexts. Cheung and Vogel [9] proposed to use the idea of quotient lattice to reduce the complexity of a Term-Document Lattice. Zhang et al. [66] discussed attribute reduction in classical formal contexts, and formulated the reduction approach by using discernibility matrices and Boolean functions. Liu et al. [40] presented a reduction method for concept lattices based on rough set theory. In [52], Wang and Zhang proposed the reduction by keeping the meet-irreducible elements. Wu et al. [57] studied knowledge reduction in formal contexts by keeping granular structure of concept lattices. Liu et al. [39] showed an efficient post-processing method to prune redundant rules in virtue of the property of Galois connection, which inherently constrains rules with respect to objects. Based on the Lukasiewicz implication, Elloumi et al. [11] gave a multi-level conceptual data reduction approach via the reduction of the object sets by keeping only the minimal rows in a formal context. Belohlavek and Vychodil [4] formulated a method to reduce the number of formal fuzzy concepts by only keeping the so-called crisply generated fuzzy concepts. Li and Zhang [37] introduced the notion of δ-reducts in a fuzzy formal context, and gave some equivalent characterizations of δ-consistent sets to determine δ-reducts. Based on fuzzy K-means clustering, Kumar and Srinivas [27] presented a method to reduce the size of a concept lattice by employing corresponding object-attribute matrix. Li et al. [38] proposed a rule-acquisition oriented framework of knowledge reduction for real decision formal contexts and formulated a reduction method by constructing a discernibility matrix and its associated Boolean function.

Both FCA and RS deal with knowledge discovery and knowledge representation on the basis of a binary relation between a set of objects and a set of attributes which is referred to as a formal context (in FCA theory) and an information system (in RS theory). As we know, FCA and RS are related and often complementary methodologies. For example, FCA can be introduced into RS theory by means of different types of formal concepts [14,61,62]. On the other hand, rough set approximation operators can be introduced into FCA by means of different types of definability [22,45,46]. The comparison and combination of FCA and RS theory may provide new approaches to data analysis and knowledge discovery. To entertain such question, much effort has been made to compare and combine these two theories in recent years [30,31,36,50,55]. In [50], Wang and Liu proposed AFS formal concept via rough set and AFS algebra which is a generalization and development to monotone concept. Lai and Zhang [30] showed that every complete fuzzy lattice can be represented as the concept lattice of a fuzzy context based on rough set theory if and only if the residuated lattice $(L,*,1)$ satisfies the law of double negation. In [60], based on the notion of homomorphisms of formal contexts, Yang and the first author of this paper discussed the invariant characters of formal contexts under homomorphisms, and reveal the relation of attributes characters and formal concepts between two formal contexts under homomorphisms and isomorphisms. By using the idea of knowledge reduction in rough set theory, Mi et al. [41] formulated a Boolean approach to calculate all reducts of a formal context via the use of discernibility function. In order to find the similar expressions with attribute reduction in rough set theory, Wang [51] developed notions and methods of attribute reduction in a formal context based on congruence relations.

For a formal context, one can consider two types of reduction: RS reduction (based on classification or decision rules) and FCA reduction (based on concepts and their hierarchy). However, what exactly is the relation between FCA and RS remains an open question.

It is worth recalling that the relations between RS reduction and FCA reduction have been discussed in the literature. For instance, based on an equivalence relation, Qi et al. [43], Wang and Zhang [49], independently discussed the relation between partition and lattice structure for a certain database and showed that each extension of formal concepts is the union of some equivalent classes. Wang and Zhang [49] presented that every equivalent class is the extent of a formal concept in an anti-chain formal context. Unfortunately, this result relies on the assumption that the formal context is anti-chain, in general, is a characteristic very difficult to meet. Wei and Qi [53], Wang and Zhang [49] showed that concept-lattice consistent set is a rough-set consistent set, but the reverse does not hold. These research results have enhanced our understanding of the relation between FCA and RS. Although some connections between FCA and RS have been revealed, but the results are insufficient. For instance, in previous literature, RS reduction and RS classification are analyzed based on an equivalent relation, which is a binary relation between objects. However, in most situations, the binary relation is partial order instead of equivalence. Hence, there are some problems need to be further considered under general relation. For instance,

1. What is the relation between classification (in partial order relation) and lattice structures for a formal context?
2. What is the relation between rough set reduction (in partial order relation) and concept lattice reduction?
3. What is the relation between attribute characteristics (in partial order relation) in rough set and concept lattice?

Inspired by the work of Qi [43], Wang and Zhang [49], Wei and Qi [53], a combination of FCA and RS theory is undertaken in the present paper to give answers to the above three questions. Differing from the previous studies in [43,49,53], we consider dominance relation instead of equivalence relation in RS model. The derived results may provide a meaningful bridge between FCA and RS.

3. Preliminaries

In this section, basic notions of FCA are briefly reviewed first. Then, we summarize the definition and approach of granular reduct in [57], which serves as a basis for the present study.

A formal context is a triplet $\mathbb{K} = (U, A, I)$, where U is a non-empty finite set of objects and A is a non-empty finite set of attributes, and I is a relation between U and A, which is a subset of the Cartesian product $U \times A$, where $(x,a) \in I$ means that object x has attribute a. For a pair of elements $x \in U$ and $a \in A$, if $(x,a) \in I$, we write xIa and read it as "object x has attribute a", or "attribute a is possessed by object x".

Let (U, A, I) be a formal context, with $X \subseteq U$ and $B \subseteq A$. The operator * is defined as:

$$X^* = \{a \in A | \forall x \in X, (x,a) \in I\},$$
$$B^* = \{x \in U | \forall a \in B, (x,a) \in I\}.$$