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Consensus model for multiple criteria group decision making under intuitionistic fuzzy environment

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ABSTRACT

Group decision making with consensus requirement is the process of reaching group consensus, ranking the feasible alternatives and selecting the best one. In this paper, we develop a methodology for fuzzy group decision making with group consensus. Firstly, each expert makes his/her judgement on each alternative with respect to multiple criteria by the intuitionistic fuzzy sets, the group preference vectors for each alternative are calculated by the formula. Secondly, the similarity measure between two intuitionistic fuzzy sets is defined to compute each expert's decision deviation, a threshold value is used to determine the decision deviation whether be acceptable. Then, based on the expert's group consensus decision information, the group matrix is obtained by weighted similarity measure. Using the ordered weight operator, the order of the alternatives is got and the best one can be easily selected. Finally, we apply our method to facility location selection problem and the other group consensus example in [3] to verify our methodology's feasibility and effectiveness.

1. Introduction

Decision making is the process of ranking feasible alternatives and selecting the best one by considering multiple criteria, it is comprised of four steps: (1) information acquisition; (2) decision-making models; (3) decision results acquisition; (4) ranking alternatives in a sequence. In some critical situation, it is not possible for a single expert to consider all the relevant aspects of a problem [1]. Therefore, the decision making process is necessary to take place with many experts from various fields. In many real life decision making problem, the group is comprised by different experts from various fields, as a diversity of ages, education backgrounds, knowledge structure and work experience, etc. It is very rare to get the same judgement on the alternatives, therefore, it is necessary to eliminate the diversity and reach group consensus. Finding solutions to group decision analysis problems [2] generally consists of two processes: a consensus process and a selection process. The consensus process is a preferable process to eliminate the non-consensus among experts, find and solve potential problems in a group decision-making process. After the consensus process, the selection process is applied to generate a group satisfactory solution. Some consensus models for group decision making can be found in the literature, Xu [3] developed an automatic approach to reach consensus among group opinions. Dong et al. presented two AHP consensus models under row geometric mean prioritization method for solving group decision-making problem in reaching consensus, the adjusted judgement matrix has better consistent index than the original judgement matrix [4]. Fu and Yang proposed an attribute weight based on feedback model for multiple attribute group decision-making with group consensus. A suggestion rule was introduced to renew the experts' assessments and a identification rule was to select the best alternatives [5]. But the above three methods are unsuitable for fuzzy multiple criteria group decision making (FMCGDM) problems with group consensus.

Owing to the complexity, fuzziness and uncertainties of the objective things, the criterion values often take the form of fuzzy information. FMCGDM as an important branch of modern decision making science, has been extensively applied in various areas: society science, management science, economics, military research, public administration, and emergency management evaluation [6]. In the last few years, many authors worked on the FMCGDM problems and gained a lot of achievements. Some related FMCGDM methods based on the extension of fuzzy sets (numbers) are proposed, there are ordered weighted aggregation operators [7], weighted geometric aggregation operators [8], TOPSIS method [9], fuzzy optimization method [10], analytic hierarchy process method [11], fuzzy hierarchical criteria group decision-making method [12], gray relational analysis method [13], and similarity

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measures [14]. As an effective method to solve intuitionistic fuzzy multicriteria group decision making problems, many studies have been done on the concepts of similarity measures between two intuitionistic fuzzy sets. On the one hand, the similarity measures were defined based on distance models, such as the Hamming distance similarity method [15], the Hausdorff distance similarity measure [16], the Euclidean distance similarity measure [17], etc. On the other hand, the intuitionistic fuzzy set was seen as a vector contains two elements, by using the vector to define the similarity measures between two intuitionistic fuzzy sets, for example, the Cosine similarity measures [18]. However, the above methods which are applied in FMCGDM are without considering group consensus requirement. Some well-known aggregation operators and fuzzy averaging from different authors have been presented in [19]. In [20], Vrana et al. introduced a new method for aggregating experts' opinions and proposed a new aggregation operator Max-Agm, based on Shannon entropy, which maximizes the agreement of experts' operators. Application of the method and the software was illustrated in a case in point study on flood risk management. So in this paper, we introduce a much simple FMCGDM methodology with group consensus requirements, the expert makes his/her judgement on the alternatives in the linguistic term, which can be converted to intuitionistic fuzzy set, we also discuss the experts who refuse to revise his/her preference information and give the suggestion on how the expert to revise his/her preference information.

The rest of this paper is set out as follows: the next section gives some basic definitions of fuzzy set and intuitionistic fuzzy set, the similarity measure between two intuitionistic fuzzy sets is also defined. The comparison with the existing similarity measures will be given in Section 3. In Section 4, we establish the model for FMCGDM with group consensus, the flowchart of the process of the methodology is also given. Facility location selection problem is solved to verify our method's feasibility and effectiveness, we also compare our method with the other consensus model [3] in Appendix. The paper ends with conclusion in Section 5.

2. Preliminaries

In this section, we introduce some basic concepts and definitions related to fuzzy set, intuitionistic fuzzy set, which will be needed in the following analysis.

2.1. The notions of fuzzy set and intuitionistic fuzzy set

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse, and $W = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of elements $x_j (j = 1, 2, \ldots, n)$, where $w_j \geqslant 0$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 1 (*Zadeh [21]*). A fuzzy set A in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ is defined as:

$$A = \{< x_j, \mu_A(x_j) > | x_j \in X\},$$

which is characterized by the membership function $\mu_A(x) : x \to [0, 1]$. Where $\mu_A(x_j)$ denotes the degree of membership of element x_j to the set A.

Let A and B be two fuzzy sets, the basic operations of fuzzy sets are defined as follows:

$$A \wedge B = \{< x_j, \mu_A(x_j) \wedge \mu_B(x_j) > | x_j \in X\};$$
$$A \vee B = \{< x_j, \mu_A(x_j) \vee \mu_B(x_j) > | x_j \in X\};$$
$$A \oplus B = \{< x_j, \mu_A(x_j) + \mu_B(x_j) - \mu_A(x_j)\mu_B(x_j) > | x_j \in X\};$$
$$A \otimes B = \{< x_j, \mu_A(x_j)\mu_B(x_j) > | x_j \in X\}.$$

It is obvious that $A \wedge B$, $A \vee B$, $A \oplus B$, $A \otimes B$ are also fuzzy sets.

Atanassov extended the fuzzy set to the intuitionistic fuzzy set (IFS) [22], shown as follows:

Definition 2. (Atanassov, [22]) An IFS A in X is given as:

$$A = \{< x_j, \mu_A(x_j), \nu_A(x_j) > | x_j \in X\},$$

which is characterized by a membership function μ_A and a non-membership function ν_A. Where

$$\mu_A : X \longrightarrow [0, 1], \quad x_j \in X \longrightarrow \mu_A(x_j) \in [0, 1],$$
$$\nu_A : X \longrightarrow [0, 1], \quad x_j \in X \longrightarrow \nu_A(x_j) \in [0, 1],$$

with the condition $\mu_A(x_j) + \nu_A(x_j) \leqslant 1$, for all $x_j \in X$.

For each IFS A in X, if $\pi_A(x_j) = 1 - \mu_A(x_j) - \nu_A(x_j), x_j \in X$, then $\pi_A(x_j)$ is called the degree of indeterminacy of x_j to A. Especially, if $\pi_A(x_j) = 0$ for each $x_j \in X$, the IFS A is reduced to a fuzzy set.

Let A and B be two IFSs, the basic operations of IFSs [23] are defined as follows:

$$\overline{A} = \{< x_j, \nu_A(x_j), \mu_A(x_j) > | x_j \in X\};$$

$$A \wedge B = \{< x_j, \mu_A(x_j) \wedge \mu_B(x_j), \nu_A(x_j) \wedge \nu_B(x_j) > | x_j \in X\};$$

$$A \vee B = \{< x_j, \mu_A(x_j) \vee \mu_B(x_j), \nu_A(x_j) \vee \nu_B(x_j) > | x_j \in X\};$$

$$A \oplus B = \{< x_j, \mu_A(x_j) + \mu_B(x_j) - \mu_A(x_j)\mu_B(x_j), \nu_A(x_j)\nu_B(x_j) > | x_j \in X\};$$

$$A \otimes B = \{< x_j, \mu_A(x_j)\mu_B(x_j), \nu_A(x_j) + \nu_B(x_j) - \nu_A(x_j)\nu_B(x_j) > | x_j \in X\};$$

$$\lambda A = \{< x_j, [1 - \mu_A(x_j)]^{\lambda}, [\nu_A(x_j)]^{\lambda} > | x_j \in X\};$$

$$A^{\lambda} = \{< x_j, 1 - [\mu_A(x_j)]^{\lambda}, 1 - [1 - \nu_A(x_j)]^{\lambda} > | x_j \in X\}.$$

It is obvious that $\overline{A}$, $A \wedge B$, $A \vee B$, $A \oplus B$, $A \otimes B$, λA and A^{λ} are also IFSs.

2.2. Some similarity measures between two intuitionistic fuzzy sets

In the vector space, there are many similarity measures, we introduce two important vector similarity measures.

Let $Y = (y_1, y_2, \ldots, y_n)$ and $Z = (z_1, z_2, \ldots, z_n)$ be two vectors of length n, where all the coordinates are positive.

The Dice similarity measure [24] is defined as follows:

$$D(Y, Z) = \frac{2YZ}{\|Y\|_2^2 + \|Z\|_2^2} = \frac{2\sum_{i=1}^{n} y_i z_i}{\sum_{i=1}^{n} y_i^2 + \sum_{i=1}^{n} z_i^2}.$$

Salton and McGill defined the cosine of the angle between two vectors as a similarity measure [25] in Definition 3.

Definition 3 (*Salton and McGill [25]*). The cosine similarity measure is defined as:

$$C(Y, Z) = \frac{YZ}{\|Y\|_2 \|Z\|_2} = \frac{\sum_{i=1}^{n} y_i z_i}{\sqrt{\sum_{i=1}^{n} y_i^2}\sqrt{\sum_{i=1}^{n} z_i^2}}.$$

Based on the Cosine similarity measure in Definition 3, Ye proposed Cosine similarity measure between two IFSs in [18], as follows:

Definition 4. Let A and B are two IFSs and contain n elements, respectively. A C-similarity measure between two IFSs A and B is proposed as follows:

$$C(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}.$$