Knowledge-Based Systems 92 (2016) 138–150

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Specification of a performance indicator using the evidential-reasoning approach

Jian Cao [a,b], Guanghua Chen [c], Mohammad Khoveyni [d], Robabeh Eslami [e], Guoliang Yang [f,*]

[a] Institute of Psychology, Chinese Academy of Sciences, Beijing 100101, China
[b] College of Humanities & Social Sciences, University of Chinese Academy of Sciences, Beijing 100039, China
[c] Donlinks School of Economics and Management, University of Science and Technology Beijing, Beijing 100083, China
[d] Department of Applied Mathematics, College of Basic Sciences, Yadegar-e-Imam Khomeini (RAH) Shahr-e-Rey Branch, Islamic Azad University, Tehran, Iran
[e] Department of Mathematics, Faculty of Technology and Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran
[f] Institute of Policy and Management, Chinese Academy of Sciences, Beijing 100190, China

A R T I C L E I N F O

Article history:
Received 10 June 2015
Revised 17 October 2015
Accepted 20 October 2015
Available online 28 October 2015

Keyword:
Data envelopment analysis
Efficient frontier
Anti-efficient frontier
The evidential-reasoning approach

A B S T R A C T

There are three primary encountered problems in classic data envelopment analysis (DEA), which they decrease the effectiveness and reliability of decision making based on the obtained information from the classic DEA. These three problems include the following issues: (1) DEA efficiency scores overestimate efficiency and they are biased; (2) In certain cases, the standard DEA models are not as useful as expected in the sense of discriminating the decision making units (DMUs); (3) Specification of the evaluated DMUs as efficient by using DEA are peculiar rather than superiority. Tackling these mentioned problems is the motivation for creating this current study. To overcome these three problems in DEA together and enhance the effectiveness and reliability of the decision-making process, this paper uses the evidential-reasoning (ER) approach to construct a performance indicator for combining the efficiency and anti-efficiency obtained by DEA and inverted DEA models, which they are used to identify the efficient and anti-efficient frontiers, respectively. Numerical simulation tests indicate that our new performance indicator is more suitable for the cases where there are relatively few DMUs to be evaluated with respect to the number of input and output indicators. Furthermore, empirical studies demonstrate that this indicator has considerably more discrimination power than that of the standard DEA models, and also it reduces overestimation and addresses peculiar DMUs, properly.

1. Introduction

Data envelopment analysis (DEA), which first introduced by Charnes et al. [13], has been widely used in productivity or performance evaluation and also the efficiency analysis of many businesses and non-profit organizations. The core idea of the classic DEA is first to identify the production frontier and then, the decision making units (DMUs) on the frontier will be regarded as efficient. Those DMUs that are not on the frontier will be compared with their peers or projections on the frontier to measure their relative efficiencies. All of the DMUs on the frontier are considered to represent the best practices and they have the same level of performance.

However, DEA efficiency scores usually overestimate the efficiency and they are biased [9]. Smith [38] argued that the classic DEA always overestimates the true efficiencies, and the main reason for the overestimation is that many inefficient units have been incorrectly classified as efficient by the classic DEA. The extent of the overestimation is dependent on the sample size and the complexity of the production process (as indicated by the number of inputs and outputs). This problem is denoted as overestimation in this paper.

Second, as we know, one of the main advantages of DEA is to allow the DMUs to have full flexibility to select their most favorable weights for their assessments to achieve the maximum efficiency scores. This full flexibility of selecting weights is important for identifying inefficient DMUs. However, this full flexibility may reduce the discrimination capacity of DEA in the sense that there are often many DMUs on the frontier, which they cannot be ranked further in the classic DEA models. Entani et al. [21] noted that the number of evaluating DMUs as efficient will increase combinationally as the dimensions of inputs and outputs increase. When there are many input and output variables and only a few DMUs, decision makers may find that most DMUs are efficient. Adler et al. [1] argued: "*Often decision-makers are interested in a complete ranking, beyond the dichotomized classification, to refine the evaluation of the units.*" This problem is denoted as discrimination in this context.

Third, Entani et al. [21] argued that some of the evaluated DMUs as efficient by using DEA are peculiar rather than superiority. In their

* Corresponding author. Tel.: +86 10 59358816.
E-mail address: glyang@casipm.ac.cn (G. Yang).

http://dx.doi.org/10.1016/j.knosys.2015.10.023
0950-7051/

example with crisp data, they showed that peculiar DMUs, which are the intersection of efficient and anti-efficient frontiers, should not be evaluated as efficient, because the widths of their efficiency intervals are very large, and other superior DMUs (identified as efficient in their example) can dominate them. This problem is denoted as dealing with peculiar DMUs. These three main encountered problems in the classic DEA models, which are based on the distances to efficient frontiers, will decrease the reliability of decision making.

There are already numerous published studies related to these three problems. However, most of these studies cannot solve the three problems together in a single framework. See Section 2 for details. This paper aims to provide a new idea to overcome some extent these three problems together including overestimation, discrimination and the handling of peculiar DMUs to rank DMUs using the evidential-reasoning approach (ER approach) (see, *e.g.*, [47,48]) to combine the efficient and anti-efficient scores. The main contribution of this paper is to consider the efficient and anti-efficient scores as two pieces of evidence and then use the ER approach based on the evidence theory to combine two pieces of evidence for each DMU to: (1) Lower the overestimation; (2) Increase the discrimination power; (3) Deal with peculiar DMUs properly within a single framework.

The remainder of this paper is organized as follows. Section 2 summarizes the related literatures for addressing the above three problems. We discuss about the efficient and anti-efficient frontiers in DEA models in Section 3. In Section 4, we first give a basic mathematical recall regarding the ER approach, and then we transform the efficiency scores or anti-efficiency scores to two pieces of evidence for combining the obtained information from both the best and worst viewpoints. Then, in this section, we conduct a numerical simulation process to test the performance of the proposed approach in this paper. In Section 5, we first provide an empirical example to illustrate the features of the ER approach, and then we perform a case study to examine the performance of the generated results from the ER approach. Finally, some conclusions are presented in Section 6.

2. Literature review

2.1. Overestimation

Banker [9] recognized in theoretical work that DEA efficiency scores overestimate efficiency and they are biased for a finite example. He argued that the efficient frontier is biased below the true efficient frontier for a finite sample size. When the sample size is small, the efficiency scores of DMUs are considerably higher than their true efficiency scores. Smith [38] reported that "*In the deterministic setting assumed here, using a convex production function, a well-specified DEA model will always overestimate efficiency. However, the extent of the overestimate is highly dependent on sample size. In effect, a larger sample size increases the possibility of encountering DMUs close to the production frontier, and therefore the DEA frontier approaches the true frontier asymptotically as sample size increases. For example, using the two input model, the average overestimate reduces from an average of 31% with samples of size 10 to just 8% as sample size increases to 80.*" Alirezaee et al. [5] showed that the high average efficiency is the result of assuming that the units in the efficient set are 100% efficient. Galagedera and Silvapulle [23] contributed to this issue by investigating the sensitivity of DEA efficiency estimates to include inappropriate and/or by omitting several important variables in a large-sample DEA model. They found that DEA tends to overestimate the efficiency in nearly all production units for constant and decreasing returns to scale (RTS) processes with irrelevant inputs.

The statistical properties of the nonparametric estimators were determined by the models developed by Kneip et al. [25] and Simar and Wilson [35], and they demonstrated that the speed of convergence of DEA estimators relies on (1) the smoothness of the unknown frontier and (2) the number of inputs and outputs relative to the number of observations. If the number of variables is relatively large, the estimators show have very low rates of convergence, and a rather large quantity of data will be needed to avoid substantial variance and very wide confidence interval estimates. To avoid the dimensionality problem, Simar and Wilson [35] suggested that the number of observations should increase exponentially with the addition of variables. According to the Simar and Wilson bootstrap results, even the simple case with a single input and single output requires at least 25 observations and preferably more than 100 for the confidence intervals of the efficiency estimator to be nearly exact. Moreover, Banker [10], Simar and Wilson [36] and Pastor et al. [31] suggested statistical tests to measure the relevance of inputs or outputs and tests for considering potentially aggregating inputs or outputs. Simar and Wilson [37] also noted when the number of inputs and outputs is large, the imprecision of the results will be reflected in large bias, large variances and wide confidence intervals. However, large samples are not generally available in practice, and researchers try to handle small multivariate data sets. Hence, it is required to find alternative methods to avoid efficiency overestimation in DEA models to some extent in the case where there are limited observations.

2.2. Discrimination

Cooper et al. [16] proposed a rule of thumb, for the number of required DMUs in DEA models, as

$$n \geq \max\{m \times s, 3(m + s)\},$$

where n is the number of DMUs, and m and s are the number of inputs and outputs, respectively. However, the rule above is sometimes violated in reality in the case of a small sample of DMUs with many input and output variables. In this case, many DMUs will often be categorized as efficient DMUs in the standard DEA models, which they are not as useful as expected in the sense of discriminating the DMUs.

Therefore, many researchers have attempted to improve the discrimination power of the standard DEA models. Podinovski and Thanassoulis [32] pointed out that there are some approaches which they can be used to improve the discrimination of DEA. It includes some simple methods (*e.g.*, the aggregation of inputs or outputs, the use of longitudinal data) and more advanced methods (*e.g.*, the use of weight restriction, production trade-offs, the use of selective proportionality between the inputs and outputs). Adler and Golany [2,3] suggested using principal components to improve discrimination in DEA with minimal loss of information. Despotis [18] introduced the global efficiency approach as a means to improve DEA discrimination power. Jenkins and Anderson [24] used the partial covariance analysis to choose a subset of variables for increasing the discrimination of DEA. Adler and Yazhemsky [4] applied Monte Carlo simulation to generalize and compare two discrimination improving methods (Principal component analysis applied to DEA and variable reduction based on partial covariance).

In our view, there are three main approaches to improve discrimination in current DEA literatures. The first approach requires preferential or prior information of decision makers to increase the discrimination ability of DEA models, *e.g.*, some scholars have developed the weights restriction [6,15,20,39] or preference change methods [28,29,51] to incorporate the decision makers' value judgments into DEA models. Although this approach can increase the discrimination power of DEA significantly, it needs more prior information on preference of decision makers. Furthermore, this approach cannot solve the problems of overestimation well. The second popular approach is the super-efficiency method, which it obtains the score of the DMU being evaluated by excluding itself from the reference set [8,40]. The super-efficiency method can also increase the discrimination power of DEA models, but it fails to address the overestimation and peculiar DMUs. Additionally, it is clear that this model uses different reference sets to evaluate the efficient DMUs and inefficient DMUs. Furthermore,