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Short Communication

IF-TODIM: An intuitionistic fuzzy TODIM to multi-criteria decision making

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ABSTRACT

The recently developed fuzzy TODIM (an acronym in Portuguese for iterative multi-criteria decision making) method using fuzzy numbers has been applied to uncertain MCDM problems with promising results. In this paper, a more general approach to the fuzzy TODIM, which takes into account the membership and the non-membership of the fuzzy information is considered. So, the fuzzy TODIM method has been extended to handle intuitionistic fuzzy information. This way, it is possible to tackle more challenging MCDM problems. Two case studies are used to illustrate and show the suitability of the developed method.

1. Introduction

Multi-criteria decision making (MCDM) problems occur in different areas of science and engineering [12]. Several research efforts have been made in order to develop new methods or to improve existing ones. Typical challenges for MCDM methods are uncertainty, risk, etc. In this sense, the theory of fuzzy sets and fuzzy logic developed by Zadeh [28] has been used to model uncertainty or lack of knowledge and applied to a variety of MCDM problems. Bellman and Zadeh [3] introduced the theory of fuzzy sets in MCDM problems as an effective approach to treat vagueness, lack of knowledge and ambiguity inherent in the human decision making process which are known as fuzzy multi-criteria decision making (FMCDM). For real world-problems the decision matrix is affected by uncertainty, which can be modeled using fuzzy numbers [7].

Another important aspect of decision making is to consider the risk attitude/preferences of the decision maker in MCDM. Prospect theory developed by Kahneman and Tversky [13] is a descriptive model of individual decision making under condition of risk. In turn, Tversky and Kahneman [21] proposed the cumulative prospect theory, which capture psychological aspects of decision making under risk. In prospect theory, the outcomes are expressed by means of gains and losses from a reference alternative [18]. The value function in prospect theory assumes a S-shape concave above the reference alternative, which reflects the aversion of risk in face of gains; and the convex part below the reference alternative reflects the propensity to risk in case of losses.

One of the first MCDM methods based on prospect theory was proposed by Gomes and Lima [9]. Despite its effectiveness and simplicity in concept, this method presents some shortcomings because of its inability to deal with uncertainty and imprecision inherent in the process of decision making. In the original formulation of TODIM (an acronym in Portuguese for Iterative Multi-criteria Decision Making), the rating of alternatives, which composes the decision matrix, is represented by crisp values. Since the TODIM method [10] is not able to handle uncertainty, Krohling and de Souza [15] proposed a fuzzy TODIM to tackle uncertain MCDM problems. A clear advantage of this method is its ability to treat uncertain information using fuzzy numbers. Recently, Fan et al. [8] have presented an extension of the TODIM method, whereas the attribute values (crisp numbers, interval numbers and fuzzy numbers) are expressed in the format of random variables with cumulative distribution functions and next the classical TODIM can be applied.

Atanasov [1] proposed a more general theory for fuzzy numbers, known as intuitionistic fuzzy numbers, which are described by a membership function and a non-membership function. In the last few years, intuitionistic fuzzy numbers have been applied to solve MCDM problems [2,27,16,17,4,5,19,22–26]. In this paper, based on the fuzzy TODIM method [15] and intuitionistic fuzzy

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numbers [1], we propose the intuitionistic fuzzy TODIM method, for short, IF-TODIM to handle uncertain MCDM problems.

The remainder of this article is organized as follows. In Section 2, some preliminary background on intuitionistic fuzzy numbers is provided. In Section 3, an intuitionistic fuzzy TODIM method is developed, which contains uncertainty in the decision matrix modeled by intuitionistic trapezoidal fuzzy numbers. In Section 4, case studies are presented to illustrate the method and the results show the feasibility of the approach. In Section 5, some conclusions and directions for future work are given.

2. Intuitionistic fuzzy multi-criteria decision making

Trapezoidal intuitionistic fuzzy numbers are commonly used for solving decision-making problems, where the available information is imprecise. Next, some basic definitions of intuitionistic fuzzy sets and fuzzy numbers are provided [1,6,11,20].

2.1. Preliminaries on intuitionistic fuzzy sets and intuitionistic fuzzy numbers

Definition 1. Let X be the universe of discourse. An intuitionistic fuzzy set $\widetilde{A}$ is characterized by a subset of X defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A : X \to [0;1]$ and $\nu_A : X \to [0;1]$ with the condition $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1 | \forall x \in X$. The numeric values $\mu_A(x)$ and $\nu_A(x)$ stands for the degree of membership and the degree of non-membership of x in A, respectively.

Definition 2. An intuitionistic trapezoidal fuzzy number $\tilde{a}$ is defined by $\tilde{a} = (a_1, a_2, a_3, a_4; \mu_{\tilde{a}}, \nu_{\tilde{a}})$ with membership function given by [22,24]:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} \cdot \tilde{\mu}_{\tilde{a}}, & a_1 \leqslant x < a_2 \\ \tilde{\mu}_{\tilde{a}}, & a_2 \leqslant x \leqslant a_3 \\ \frac{a_4-x}{a_4-a_3} . \tilde{\mu}_{\tilde{a}}, & a_3 < x \leqslant a_4 \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

while the non-membership function is given by:

$$\nu_{\tilde{a}}(x) = \begin{cases} \frac{(a_2-x)+\tilde{\nu}_{\tilde{a}}(x-a_1)}{a_2-a_1}, & a_1 \leqslant x < a_2 \\ \tilde{\nu}_{\tilde{a}} & a_2 \leqslant x \leqslant a_3 \\ \frac{(x-a_3)+\tilde{\nu}_{\tilde{a}}(a_4-x)}{a_4-a_3}, & a_3 < x \leqslant a_4 \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

The values $\tilde{\mu}_{\tilde{a}}$ and $\tilde{\nu}_{\tilde{a}}$ represent the maximum value of membership degree and non-membership degree of $\tilde{a}$, respectively. For instance, consider the intuitionistic trapezoidal fuzzy number (ITFN) $\langle VG; 0.6, 0.3\rangle = \langle 0.5, 0.75, 0.95, 1; 0.6, 0.3\rangle$. In this case, a decision maker not only assess the rating of an alternative by using the linguistically defined trapezoidal fuzzy number (TFN) *VG (Very Good)* but also provides the degree of membership and non-membership, 0.6 and 0.3 respectively.

Definition 3. Let a trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$, then its expected value is calculated as $I(\tilde{a}) = [(a_1 + a_2 + a_3 + a_4) \cdot (1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}})]/8$. In addition, definitions for $S(\tilde{a}) = I(\tilde{a}) \cdot (\tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}})$ and $H(\tilde{a}) = I(\tilde{a}) \cdot (\tilde{\mu}_{\tilde{a}} + \tilde{\nu}_{\tilde{a}})$ are presented, which are known as score function and accuracy function, respectively [22].

Definition 4. Let two intuitionistic trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$, then [22]:

If $S(\tilde{a}) > S(\tilde{b})$ then $\tilde{a} > \tilde{b}$.
If $S(\tilde{a}) = S(\tilde{b})$ and If $H(\tilde{a}) = H(\tilde{b})$ then $\tilde{a} = \tilde{b}$.
If $S(\tilde{a}) = S(\tilde{b})$ and $H(\tilde{a}) > H(\tilde{b})$ then $\tilde{a} > \tilde{b}$.

Definition 5. Let two intuitionistic trapezoidal fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4; \tilde{\mu}_{\tilde{a}}, \tilde{\nu}_{\tilde{a}})$ and $\tilde{b} = (b_1, b_2, b_3, b_4; \tilde{\mu}_{\tilde{b}}, \tilde{\nu}_{\tilde{b}})$, then the distance between them is calculated as [22]:

$$\begin{aligned} d(\tilde{a}, \tilde{b}) = &\frac{1}{8}\big[|(1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}}) \cdot a_1 - (1 + \tilde{\mu}_{\tilde{b}} - \tilde{\nu}_{\tilde{b}}) \cdot b_1| \\ &+ |(1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}}) \cdot a_2 - (1 + \tilde{\mu}_{\tilde{b}} - \tilde{\nu}_{\tilde{b}}) \cdot b_2| \\ &+ |(1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}}) \cdot a_3 - (1 + \tilde{\mu}_{\tilde{b}} - \tilde{\nu}_{\tilde{b}}) \cdot b_3| \\ &+|(1 + \tilde{\mu}_{\tilde{a}} - \tilde{\nu}_{\tilde{a}}) \cdot a_4 - (1 + \tilde{\mu}_{\tilde{b}} - \tilde{\nu}_{\tilde{b}}) \cdot b_4|\big]. \end{aligned} \tag{3}$$

For instance, consider $\tilde{a} = (VL; 0.8, 0.1) = (0.1, 0.2, 0.3, 0.4; 0.8, 0.1)$ and $\tilde{b} = (EH; 0.7, 0.2) = (0.7, 0.8, 0.9, 0.95; 0.7, 0.2)$, where VL and EH are linguistic definitions of the trapezoidal fuzzy numbers, *Very Low* and *Extremely High*, respectively. The distance between them is $d(\tilde{a}, \tilde{b}) = 0.4156$.

2.2. Decision making problem with uncertain decision matrix

Let us consider the fuzzy decision matrix A, which consists of alternatives and criteria, described by:

$$A = \begin{array}{c} \\ A_1 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \dots & C_n \end{array} \\ \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{pmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are alternatives, $C_1, C_2, \ldots, C_n$ are criteria, $\tilde{x}_{ij}$ intuitionistic trapezoidal fuzzy numbers that indicates the rating of the alternative A_i with respect to criterion C_j. The weight vector $W = (w_1, w_2, \ldots, w_n)$ composed of the individual weights w_j ($j = 1, \ldots, n$) for each criterion C_j satisfying $\sum_{j=1}^{n} w_j = 1$.

In the following section, the method is presented.

2.3. IF-TODIM – An intuitionistic fuzzy TODIM method

For information on the TODIM method the reader is referred to Gomes and Rangel [10]. The intuitionistic fuzzy TODIM method, for short, IF-TODIM, which is an extension of the fuzzy TODIM method [15], is described in the following steps:

Step 1: The criteria are normally classified into two types: *benefit* and *cost*. The intuitionistic trapezoidal fuzzy-decision matrix $\widetilde{A} = [\tilde{x}_{ij}]_{mxn}$ with $i = 1, \ldots, m$, and $j = 1, \ldots, n$ is normalized which results the correspondent fuzzy decision matrix $\widetilde{R} = [\tilde{r}_{ij}]_{mxn}$. The fuzzy normalized value $\tilde{r}_{ij}$ is calculated as:

$$r_{ij}^k = \frac{\max\left(a_{ij}^4\right) - a_{ij}^k}{\max_i\left(a_{ij}^4\right) - \min_i a_{ij}^1} \text{ with } k = 1, 2, 3, 4 \quad \text{for cost criteria} \tag{4a}$$

$$r_{ij}^k = \frac{a_{ij}^k - \min\left(a_{ij}^1\right)}{\max_i\left(a_{ij}^4\right) - \min_i a_{ij}^1} \text{ with } k = 1, 2, 3, 4 \quad \text{for benefit criteria} \tag{4b}$$

We denote a_k of $a_{ij} = \{(a_1, a_2, a_3, a_4)\}$ as a_{ij}^k, e.g., $a_{ij}^3 = a_3$.

Step 2: Calculate the dominance of each alternative $\widetilde{R}_i$ over each alternative $\widetilde{R}_j$ using the following expression: