Knowledge-Based Systems 30 (2012) 115–120

Contents lists available at SciVerse ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

MADM method based on cross-entropy and extended TOPSIS with interval-valued intuitionistic fuzzy sets

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ARTICLE INFO

Article history:
Received 20 December 2010
Received in revised form 19 December 2011
Accepted 4 January 2012
Available online 12 January 2012

Keywords:
Multiattribute decision making
Interval-valued intuitionistic fuzzy set
Cross-entropy
Mathematical programming
TOPSIS

ABSTRACT

Many authors have investigated multiattribute decision making (MADM) problems under interval-valued intuitionistic fuzzy sets (IVIFSs) environment. This paper presents an optimization model to determine attribute weights for MADM problems with incomplete weight information of criteria under IVIFSs environment. In this method, a series of mathematical programming models based on cross-entropy are constructed and eventually transformed into a single mathematical programming model to determine the weights of attributes. In addition, an extended technique for order preference by similarity to ideal solution (TOPSIS) is suggested to ranking all the alternatives. Furthermore, an illustrative example is provided to compare the proposed approach with existing methods. Finally, the paper concludes with suggestions for future research.

1. Introduction

Since the introduction of fuzzy set (FS) theory by Zadeh [1], many research achievements have been made to enrich the FS theory. Interval valued fuzzy set [2], intuitionistic fuzzy set (IFS) [3] and vague set [4] are all well known generalizations of fuzzy set and are extensively applied in many fields. Later IFS was further extended to interval-valued intuitionistic fuzzy set (IVIFS) by Atanassov and Gargov [5].

Owing to the advantage of dealing with uncertain information, many theories and methods on IVIFSs have been put forward and used to solve fuzzy MADM problems. For example, a series of arithmetic and geometric aggregation operators [6–10] were proposed to aggregate interval-valued intuitionistic fuzzy information. For the purpose of ranking IVIFSs, some literature [7,8,11–13] constructed various kinds of score functions or accuracy functions in succession. Correlation coefficients [14–16] were calculated and applied to measure the correlation between alternative and ideal alternative under IVIFSs environment. Furthermore, TOPSIS was also suggested by [16–19] in MADM under IVIFSs environment. In addition, mathematical programming models were established by [12,17,20–22] to determine attribute weights or calculate the relative closeness coefficient intervals of alternatives to the ideal solution. However, there exists little investigation [23] on the application of entropy theory under IVIFSs environment and attention paid to avoiding information loss is not always enough in the process of information aggregation. In this paper, a programming model based on cross-entropy is defined with incomplete weight information under IVIFSs environment, which can be used to obtain the attribute weights. The rest of this paper is organized as follows. In Section 2, a brief introduction to the basic knowledge of IFSs and IVFSs is provided. In Section 3, a mathematical programming model is established to determine the attribute weight with incomplete weight information of criteria and TOPSIS is extended to determine the ranking order of all the alternatives for MADM problems under IVIFSs environment. In Section 4, an illustrative example is given to present the optimization model. Section 5 gives the conclusions and suggestions for future research.

2. Preliminaries

In the following, some basic knowledge related to IFSs and IVIFSs is introduced.

Definition 1 [4,24]. Let a set $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An IFS A in X is defined as $A = \{(x_i, \mu_A(x_i), \upsilon_A(x_i)) | x_i \in X\}$, where the functions $\mu_A(x_i)$: $X \to [0,1]$, $\upsilon_A(x_i)$: $X \to [0,1]$, with the condition $0 \leqslant \mu_A(x_i) + \upsilon_A(x_i) \leqslant 1$ for any $x_i \in X$.

The number $\mu_A(x_i)$ and $\upsilon_A(x_i)$ stand for the degree of membership and nonmembership of the element x_i to A, respectively.

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doi:10.1016/j.knosys.2012.01.003

Atanassov defined $\pi_A(x_i)$ as hesitancy degree of x_i to A: $\pi_A(x_A(x_i) = 1 - \mu_A(x_i) - \upsilon_A(x_i)$. Obviously, if $\pi_A(x_i) = 0$, IFS A is reduced to a fuzzy set. The complementary set A^C of an IFS A is defined as: $A^C = \{(x_i, \upsilon_A(x_i), \mu_A(x_i)) | x_i \in X\}$.

Definition 2 [25]. For two IFSs A and B, $I_{IFS}(A,B)$ is the intuitionistic fuzzy cross-entropy between A and B:

$$I_{IFS}(A,B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\ln\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)} + \upsilon_A(x_i)\ln\frac{2\upsilon_A(x_i)}{\upsilon_A(x_i)+\upsilon_B(x_i)}\right]. \tag{1}$$

$I_{IFS}(A,B)$ denotes the degree of discrimination of A from B, which can also be called discrimination information for IFSs. In view of symmetry, a symmetric measure is defined as follows:

$$D_{IFS}(A,B) = I_{IFS}(A,B) + I_{IFS}(B,A). \tag{2}$$

$D_{IFS}(A,B)$ is called a symmetric discrimination information measure for IFSs. It can easily be verified that $D_{IFS}(A,B) \geqslant 0$ and $D_{IFS}(A,B) = 0$ if and only if $A = B$.

Usually, the weight of the element $x_i \in X$ should be taken into account, so the following intuitionistic fuzzy weighted cross-entropy and weighted degree of discrimination of A from B can be defined as follows.

Definition 3. Assume the weight of the element $x_i \in X = \{x_1, x_2, \ldots, x_n\}$ is $w_i (i = 1, 2, \ldots, n)$, where $\sum_{i=1}^{n} w_i = 1$. For two IFSs A and B, $I_{wIFS}(A,B)$ is the intuitionistic fuzzy weighted cross-entropy between A and B:

$$I_{wIFS}(A,B) = \sum_{i=1}^{n} w_i\left[\mu_A(x_i)\ln\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)} + \upsilon_A(x_i)\ln\frac{2\upsilon_A(x_i)}{\upsilon_A(x_i)+\upsilon_B(x_i)}\right]. \tag{3}$$

$D_{wIFS}(A,B)$ denotes the weighted degree of discrimination between A and B:

$$D_{wIFS}(A,B) = I_{wIFS}(A,B) + I_{wIFS}(B,A). \tag{4}$$

Definition 4 [5]. Let a set $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An IVIFS $\widetilde{A}$ in X is defined as: $\widetilde{A} = \{(x_i, \tilde{\mu}_{\widetilde{A}}(x_i), \tilde{\upsilon}_{\widetilde{A}}(x_i)) | x_i \in X\}$, where $\tilde{\mu}_{\widetilde{A}}(x_i) \subseteq [0, 1]$, $\tilde{\upsilon}_{\widetilde{A}}(x_i) \subseteq [0, 1]$ are closed intervals and stand for the degree of membership and nonmembership of the element x_i to $\widetilde{A}$, respectively. Their lower and upper limits are expressed as $\tilde{\mu}^L_{\widetilde{A}}(x_i)$, $\tilde{\mu}^U_{\widetilde{A}}(x_i)$, $\tilde{\upsilon}^L_{\widetilde{A}}(x_i)$ and $\tilde{\upsilon}^U_{\widetilde{A}}(x_i)$, respectively. And then, the IVIFS $\widetilde{A}$ can be denoted by:

$$\widetilde{A} = \left\{\left(x_i, \left[\tilde{\mu}^L_{\widetilde{A}}(x_i), \tilde{\mu}^U_{\widetilde{A}}(x_i)\right], \left[\tilde{\upsilon}^L_{\widetilde{A}}(x_i), \tilde{\upsilon}^U_{\widetilde{A}}(x_i)\right]\right) | x_i \in X\right\}, \tag{5}$$

where $\tilde{\mu}^U_{\widetilde{A}}(x_i) + \tilde{\upsilon}^U_{\widetilde{A}}(x_i) \leqslant 1$. Similarly, the hesitancy interval of x_i to $\widetilde{A}$ is defined: $\tilde{\pi}_{\widetilde{A}}(x_i) = \left[\tilde{\pi}^L_{\widetilde{A}}(x_i), \tilde{\pi}^U_{\widetilde{A}}(x_i)\right] = \left[1 - \tilde{\mu}^U_{\widetilde{A}}(x_i) - \tilde{\upsilon}^U_{\widetilde{A}}(x_i),\right.$ $\left.1 - \tilde{\mu}^L_{\widetilde{A}}(x_i) - \tilde{\upsilon}^L_{\widetilde{A}}(x_i)\right]$.

Especially, if $\tilde{\mu}^L_{\widetilde{A}}(x_i) = \tilde{\mu}^U_{\widetilde{A}}(x_i)$ and $\tilde{\upsilon}^L_{\widetilde{A}}(x_i) = \tilde{\upsilon}^U_{\widetilde{A}}(x_i)$, IVIFS $\widetilde{A}$ is reduced to an IFS. The complementary set A^C of an IFS A is defined as: $\widetilde{A}^C = \{(x_i, \tilde{\upsilon}_{\widetilde{A}}(x_i), \tilde{\mu}_{\widetilde{A}}(x_i)) | x_i \in X\}$.

Bustince and Burillo [30] put forward an operator, $H_{p,q}$, which can transform each IVIFS into an IFS.

Definition 5 [30]. Let $p,q \in [0,1]$ be two fixed numbers, any IVIFS denoted by Eq. (5) can be transformed into an IFS by the operator $H_{p,q}$:

$$H_{p,q}(\widetilde{A}) = \left\{\left(x_i, \tilde{\mu}^L_{\widetilde{A}}(x_i) + pW_{\tilde{\mu}_{ij}}(x_i), \tilde{\upsilon}^L_{\widetilde{A}}(x_i) + qW_{\tilde{\upsilon}_{ij}}(x_i)\right) | x_i \in X\right\}, \tag{6}$$

where $W_{\tilde{\mu}_{ij}}(x_i) = \tilde{\mu}^U_{\widetilde{A}}(x_i) - \tilde{\mu}^L_{\widetilde{A}}(x_i)$ and $W_{\tilde{\upsilon}_{ij}}(x_i) = \tilde{\upsilon}^U_{\widetilde{A}}(x_i) - \tilde{\upsilon}^L_{\widetilde{A}}(x_i)$. To get the ranking order among intervals, Xu and Da [26] introduced the concept of likelihood. Suppose that $a = [a^L, a^U]$ and $b = [b^L, b^U]$ be any two intervals, where $0 \leqslant a^L \leqslant a^U$ and $0 \leqslant b^L \leqslant b^U$. If $a^L = a^U$, a will be degenerated to a real number.

Definition 6 [26]. For any two real numbers a and b, the likelihood of $a > b$ is defined as follows

$$p(a > b) = \begin{cases} 1, & a > b, \\ 0, & a \leqslant b. \end{cases}$$

If a and b are two intervals, the likelihood of $a \geqslant b$ is expressed as follows

$$p(a \geqslant b) = \max\left\{1 - \max\left\{\frac{b^U - a^L}{L(a) + L(b)}, 0\right\}, 0\right\},$$

where $L(a) = a^U - a^L$ and $L(b) = b^U - b^L$.

The likelihood of $a \geqslant b$ has the following properties:

(1) $p(a \geqslant b) = p(b \geqslant a) = 0.5$ if $p(a \geqslant b) = p(b \geqslant a)$;
(2) Complementarity: $p(a \geqslant b) + p(b \geqslant a) = 1$;
(3) $p(a \geqslant b) = 0$ if $a^U \leqslant b^L$;
(4) $p(a \geqslant b) = 1$ if $a^L \geqslant b^U$;
(5) Transitivity: for any intervals a, b and c, $p(a \geqslant c) \geqslant p(b \geqslant c)$ if $a \geqslant b$.

In addition, some main operations for intervals [33] can be expressed as follows:

$$a + b = [a^L + b^L, a^U + b^U], \tag{7}$$

$$\lambda a = [\lambda a^L, \lambda a^U], \tag{8}$$

where λ is a nonnegative real number.

Definition 7 [30]. Let $\widetilde{A} = \{(x_i, [a_i, b_i], [c_i, d_i]) | i = 1, 2, \ldots, n\}$ and $\widetilde{B} = \{(x_i, [a_i', b_i'], [c_i', d_i']) | i = 1, 2, \ldots, n\}$ be IVIFSs, the information energy of $\widetilde{A}$ is defined as follows:

$$E(\widetilde{A}) = \sum_{i=1}^{n} \frac{a_i^2 + b_i^2 + c_i^2 + d_i^2}{2}.$$

$C(\widetilde{A}, \widetilde{B})$ and $K(\widetilde{A}, \widetilde{B})$ are correlation and correlation coefficient of IVIFSs $\widetilde{A}$ and $\widetilde{B}$, respectively:

$$C(\widetilde{A}, \widetilde{B}) = \frac{1}{2}\sum_{i=1}^{n}(a_i a_i' + b_i b_i' + c_i c_i' + d_i d'),$$
$$K(\widetilde{A}, \widetilde{B}) = \frac{C(\widetilde{A}, \widetilde{B})}{\sqrt{E(\widetilde{A})E(\widetilde{B})}}. \tag{9}$$

3. MADM method based on cross-entropy and extended TOPSIS

This section puts forward a framework for determining attribute weights and ranking order for all the alternatives with incomplete weight information under IVIFSs environment.

3.1. Problem description

Assume that $C = \{C_1, C_2, \ldots, C_n\}$ be the set of attributes and $A = \{A_1, A_2, \ldots, A_m\}$ be an alternative set, which consists of m non-inferior alternatives (An alternative is non-inferior if there exists no other alternative which can yield an improvement in one attribute, without causing a degradation in another). Let