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Algorithms for computing symbolic representations of basic *e*-sums and their application to composites

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A R T I C L E I N F O

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A B S T R A C T

The main goal of the paper is to present an algorithm to realize the theoretical method by Berlyand and Mityushev (2001, 2005) for computations of the effective conductivity of fibrous composites expressed in therms of the linear combinations of the basic *e*-sums. Algorithms are presented in Mathematical Pseudo-language with examples in *Mathematica*®. A method for simplifying the basic *e*-sums by reducing *mirror* terms is presented. Developed algorithms are applied to the optimal packing problem of disks in the plane.

1. Introduction

The study of both mathematical theory and the algorithms presented in this paper have origin and application in the theory of composites. Composites are examples of random heterogeneous materials. Such a material is composed of different phases (component materials), distinguishable on the microscopic scale. Each phase has the *conductivity* property (either *thermal* or *electrical*). Heterogeneous material can be viewed as a continuum on the macroscopic scale (i.e. homogeneous material) with the *effective conductivity* (EC) determined as the overall macroscopic property of the material.

Let us consider a model of the polydispersed fiber composite when unidirectional fibres are randomly embedded in a matrix (see Fig. 1). One can reduce polydispersed fibrous composite model, when the direction of heat flux (or electric flux) is perpendicular to the direction of fibres, to the

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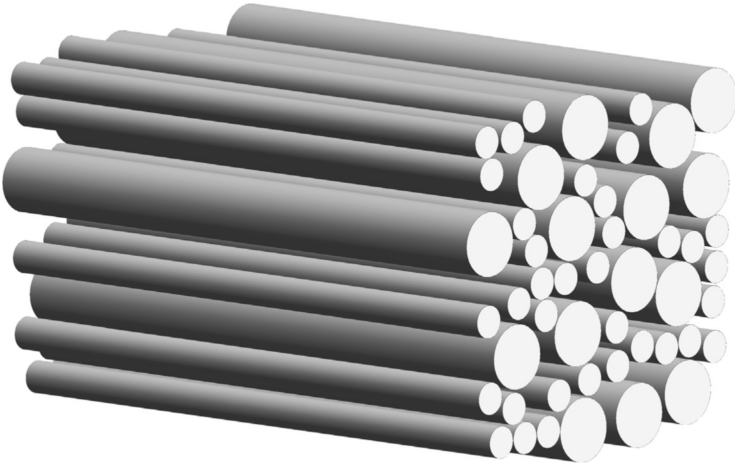

Fig. 1. Model of a polydispersed fibrous composite.

two-dimensional random composite material. Hence, the inclusions are modelled by nonoverlapping disks (a cross-section perpendicular to the direction of fibres).

In recent years applications of *symbolic approximation* and *asymptotic methods* to material sciences have noticeably grown (Andrianov et al., 2002; Berlyand et al., 2013; Manevitch et al., 2002). In wide class of problems, symbolic models approach yields results in more convenient form than in pure numerical approximations. Some constructive methods can be considered as ready-to-use symbolic algorithms solving particular problems. These methods require optimization and detailed analysis from computational point of view. One can find examples of such new algorithms arising in the study of effective conductivity of composites in Berlyand and Mityushev (2001, 2005), where the closed form representation for the EC of 2D random material with nonoverlapping circular inclusions of two different sizes was presented. The representation has the form of infinite series which coefficients are so-called B_q *coefficients*. The very same paper provides the constructive formulae for B_q in terms of elliptic functions. Unfortunately, application of those formulas brings computational difficulties for higher approximations of EC, hence one can obtain only a few first B_q coefficients by hand. Once computed symbolic expression of B_q coefficient may be rewritten in much simpler form in terms of another mathematical objects named *e-sums*, introduced by Mityushev (2006) as multidimensional discrete convolutions of elliptic functions. In fact, B_q coefficients appear to be the linear combinations of e-sums. However, the algorithm for generating B_q directly in terms of e-sums does not exists.

An obvious application of modelling composite materials is developing new materials by studying relations between their structure and properties. Another, relatively new and interesting application of EC formula, where theory of composites meets computational geometry, is solving optimal packing problems (Mityushev, 2014; Mityushev and Rylko, 2012). The more recent application comes with the paper by Mityushev and Nawalaniec (2015), where the e-sums where considered as geometrical parameters that perform the same role in description of microstructure as the n-point correlation functions. This allowed to perform systematic investigations of dynamically changing structures. In such investigation it is crucial to specify all e-sums included in a particular coefficient B_q (so-called *basic e-sums of order q*).

The main goal of this paper is to construct an algorithm for computing symbolic form of B_q, using e-sum notation only. It will be possible for scientists in material science to calculate previously unreachable approximation of EC formula. The paper also focuses on developing an algorithm for reducing a given expression of B_q coefficient in terms of e-sums.

The present paper is organized as follows. In Section 2, we review the background material on EC of composites and the optimal packing problem. In Section 3, we provide a self-contained mathematical statements of the problems that will be tackled in the subsequent sections. In Section 4, we provide an algorithm that solve the first problem. In Section 5, we provide an algorithm that