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Continuous detection of the variations of the intersection curve of two moving quadrics in 3-dimensional projective space

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We propose a symbolic algorithm for detecting the variations in the topological and algebraic properties of the intersection curve of two quadratic surfaces (QSIC) that are moving or deforming in $\mathbb{PR}^3$ (real projective 3-space). The core of our algorithm computes all the critical instants when the QSIC changes type using resultants and Jordan forms. These critical instants partition the time axis into intervals within which the QSIC is invariant. The QSIC at the computed critical instants and within the time intervals can both be exactly determined using symbolic technique. Examples are provided to illustrate our algorithm.

1. Introduction

Quadrics are the simplest curved surfaces which have both concise algebraic representations and elegant geometric properties. Hence quadrics have widely been used in CAD/CAM and industrial manufacture, where 3D shapes are frequently defined by piecewise quadrics (Wang, 2002; Yan et al., 2012). The intersection curve of two quadrics (QSIC) has attracted particular attention since it contributes to the boundary detection of 3D shapes defined by quadric patches in geometric

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modeling or industry design (Levin, 1979; Wang et al., 2003; Dupont et al., 2008a, 2008b, 2008c; Tu et al., 2009).

There is plenty of work on the classification of the QSIC. In algebraic geometry, the QSIC morphology is usually classified in $\mathbb{PC}^3$, complex projective 3-space. For example, Bromwich (1906) accomplishes this classification by means of Segre characteristic, which is defined by the order of Jordan forms associated with the roots of the characteristic polynomial of the two given quadrics, without distinguishing between real and imaginary roots. Hence in $\mathbb{PR}^3$, real projective 3-space, different QSICs might correspond to the same Segre characteristic and therefore cannot be further distinguished by Bromwich (1906).

To study the QSIC in $\mathbb{PR}^3$, two very significant works appear almost simultaneously. One is Dupont et al. (2008a, 2008b, 2008c), which presents an exact algorithm on parameterizing the QSIC in $\mathbb{PR}^3$. Their output parametrization of the QSIC is proved to be near optimal in the generic case when the QSIC is a smooth quartic curve, and is polynomial whenever possible for singular cases of QSIC, including a singular quartic curve, a cubic with a line, and two conics. Tu et al. (2009) tackled the same problem by directly classifying QSICs in $\mathbb{PR}^3$ using signature sequences, a technique finer than the Segre characteristics, which further takes the real roots and the Jordan forms associated with these real roots of the characteristic polynomial into consideration. Moreover, their classification distinguishes between QSICs both by their topological properties and the algebraic properties, which is slightly finer than morphology. As for computation, Tu et al. (2009) also provide an algorithm with rational arithmetic for computing the signature sequence and then determining the type of the QSIC.

Our work can be seen as a deeper extension of collision detection, which detect the variations in the algebraic property and topology (brief as 'type') of QSIC of two quadrics that are moving or deforming. Collision detection is widely applied in computer graphics, computer animation, robotics and computational physics. The essence of collision detection is to detect the instants when the intersection curve of the target objects changes from imaginary to real, or conversely. For related work, see Ju et al. (2001), Rimon and Boyd (1997), Shiang et al. (2000), Wang et al. (2004), Choi et al. (2006, 2009), Jia et al. (2011). An algebraic approach on continuous collision detection of two moving ellipsoids is given by Jia et al. (2011).

The objects considered in our paper are two quadrics that are moving or deforming in $\mathbb{PR}^3$. Our target is to detect the variations of their QSIC in $\mathbb{PR}^3$: We consider the QSIC variation from a slightly finer aspect than the common concept of morphology, i.e., we distinguish QSICs by their topological properties and algebraic properties, including singularities, the number of components, and the degree of each of the irreducible components. For example, we distinguish a simple real loop from a double loop; we distinguish a simple real loop from a real loop with a cusp; we also distinguish a non-degenerate QSIC with two disconnected components from a reducible QSIC with two disconnected conics. Our approach to distinguishing the type of the QSICs agrees exactly with that of Tu et al. (2009).

Our two moving quadrics are given by $\mathcal{A} : X^T A(t) X = 0$ and $\mathcal{B} : X^T B(t) X = 0$, where $X = (x, y, z, w)^T \in \mathbb{PR}^3$ and $A(t), B(t)$ are 4×4 matrices whose elements are polynomials in $\mathbb{R}[t]$. The *characteristic function* associated with $\mathcal{A}$ and $\mathcal{B}$ is defined by

$$f(\lambda, t) = \det(\lambda A(t) - B(t)), \tag{1}$$

where $\lambda \in \mathbb{R}$, hence $f(\lambda, t)$ is either a bivariate polynomial in λ, t of degree at most four in λ, or a univariate polynomial of degree at most four in λ, or vanishes identically. The moving quadric pencil $\lambda A(t) - B(t)$ is said to be *non-degenerate* if $f(\lambda, t) \not\equiv 0$, otherwise the pencil is said to be *degenerate*. The moving quadric pencil is degenerate if and only if at an arbitrary instant t the two quadrics $\mathcal{A}$ and $\mathcal{B}$ are two singular quadrics sharing a singular point or a double line. For example, $\mathcal{A}(t)$ and $\mathcal{B}(t)$ are two cones sharing the same vertex but both rotating in their own way around the vertex. Analyzing degenerate pencils is a relatively simple task and can be treated in a way quite similar to the analysis of non-degenerate pencils that we are going to describe. Hence throughout the paper we shall assume that $f(\lambda, t) \not\equiv 0$.

Our approach is twofold: First we detect all the critical instants at which the QSIC changes, which is proved to be equivalent to detecting the instants when the Segre characteristic of the quadric pencil $\lambda A(t) - B(t)$ changes. This step is the core of the whole paper, in which we separately treat two