Knowledge-Based Systems 24 (2011) 554–561

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Research on the multi-attribute decision-making under risk with interval probability based on prospect theory and the uncertain linguistic variables

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ARTICLE INFO

Article history:
Received 2 July 2010
Received in revised form 21 January 2011
Accepted 25 January 2011
Available online 1 February 2011

Keywords:
Prospect theory
Interval probability
Uncertain linguistic variables
Risk decision-making
Multi-attribute decision-making

ABSTRACT

With respect to risk decision making problems with interval probability in which the attribute values take the form of the uncertain linguistic variables, a multi-attribute decision making method based on prospect theory is proposed. To begin with, the uncertain linguistic variables can be transformed into the trapezoidal fuzzy number, and the prospect value function of the trapezoidal fuzzy number based on the decision-making reference point of each attribute and the weight function of interval probability can be constructed; then the prospect value of attribute for every alternative is calculated through prospect value function of the trapezoidal fuzzy number and the weight function of interval probability, and the weighted prospect value of alternative is acquired by using weighted average method according to attribute weights, and all the alternatives are sorted according to the expected values of the weighted prospect values; Finally, an illustrate example is given to show the decision-making steps, the influence on decision making for different parameters of value function and different decision-making reference point, and the feasibility of the method.

1. Introduction

Multiple attribute decision making (MADM) has been extensively applied to various areas, such as society, economics, management, military and engineering technology. For example, investment decision-making, project evaluation, economic evaluation, personnel evaluation etc. [14,18,20,21,26,30]. The decision makers often give the evaluation information as the linguistic terms directly for these decision problems. For example, The decision makers can use 'Very Poor', 'Poor', 'Fair', 'Good', and 'Very Good' to evaluate the automotive performance. However, due to the more fuzzy and uncertain of the decision making environment, the decision makers frequently use the uncertain linguistic to express the evaluation information. For example, decision makers can only give an evaluation value which is between 'Fair' and 'Good' for automotive performance, i.e., it is superior to 'Fair' and inferior to 'Good'. Therefore, the study on the MADM problems based on uncertain linguistic variables has very important values on theoretical and practical application. In addition, in the real decision making, the decision makers sometimes face the uncertain condition. The attribute values of decision problems are random variables and they can be changed based on the natural status. Decision makers don't determine the specific status, but they can determine all the possible status, and they can quantify this randomness by setting up the probability distribution. These decision making problems are called risk MADM [34]. Owing to the complexity of the object things, the decision makers are hard to determine the precise probability of occurrence, so we often use the interval number to express the probability, which is better to describe the decision making problems. It can be seen that the study on the risk MADM problems with interval probability and attribute values taking the form of the uncertain linguistic variables has the important values on theoretical and practical application.

The main studies on the risk MADM problems are shown as follows: Yu et al. [34] studied the risk MADM problems in which the attribute weights were unknown and the attribute values were real numbers, and set up the related mathematical model. Luo and Liu [17] studied the risk MADM problems in which the attribute weights were completely unknown and attribute values were interval numbers, and set up two algorithms, grey fuzzy relationship method and two-basic-point method. Liu and Guan [15] proposed a grey correlation ranking method to solve risk MADM problems with weight unknown and attribute values as continuous random variables on bounded intervals. Jin et al. [10] proposed a projection method to solve risk MADM problems with attributes value as continuous random variables on bounded intervals. Wang

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doi:10.1016/j.knosys.2011.01.010

and Ren [28] discussed risk MADM problems with weight information incomplete and criteria values in the form of normally distributed random variables, and then developed a multiple criteria decision making method based on WC-OWA operator. The risk MADM problems, in which the attribute values take the form of linguistic variables or uncertain linguistic variables, have not been studied in these researches.

The main studies on the MADM problems, in which attribute values take the form of linguistic variables or uncertain linguistic variables, are shown as follows: Zadeh [35] proposed the concept of a linguistic variable. Xu [32] presented an interactive procedure for linguistic multiple attribute decision making, in which the weight information was incomplete and the attribute values took the form of linguistic variable. Dong and Xiao [6] studied the group decision-making problems with natural linguistic evaluation information. Firstly, linguistic evaluation values were transformed into triangular fuzzy numbers, then the triangular fuzzy numbers were aggregated, finally, the best alternative was selected based on the aggregation results. Noor-E-Alam et al. [19] developed a computing tool which can evaluate the supplier by taking the opinion of expert as a linguistic value in a fuzzy form and incorporating the uncertainty measure. Shamsuzzaman et al. [23] proposed a fuzzy-set-AHP approach for selecting the best-ranked flexible manufacturing system from a number of feasible alternatives. Fuzzy sets were employed to recognize the selection criteria as linguistic variables rather than numerical ones. Wei and Liu [29] proposed a TOPSIS method to solve MADM problems in which attribute values took the form of uncertain linguistic variables, and to use for high-technology project investment evaluation. Liu et al. [16] proposed an uncertain linguistic weighted C-EOWA (ULWC-EOWA) operator to solve these problems. The multiple attribute decision making problems with risk decision-making information have not been studied in these researches.

To date, the multiple attribute decision-making models and methods introduced above are based on rational choice model using expected utility theory, for example, the choice can be made by the least risky or the most utility etc. expected utility theory from the perspective of the logic and reasoning illustrates that people should how to choose or decision, which is an entirely rational choice course of action. However, in the actual decision-making process, people often are not fully rational decision-making, the actual decision making behaviors depart from the predictions of expected utility theory obviously. Simon [24] proposed "bounded rationality" principle, and he thought that people's decision-making only had the limited rationality; Kahneman and Tversky [12] collected many studies of the individual behavior based on Simon's "bounded rationality" by surveys and testing, and they found that the people's judgments and decisions of the actual behavior under uncertainty environment departed form the predictions of the expected utility theory, so they proposed prospect theory in 1979. Prospect Theory is a descriptive model of individual decision making under conditions of risk [11,12,25]. To be clear, the foundations for substantive propositions behind prospect theory are empirical and experimental in nature. On the basis of Kahneman and Tversky' researches [12], many researchers further studied the parameters of the value function and weight function based on individual action experiments [1,3,22]. On the other hand, as the leading behavioral model of decision making under risk, Prospect theory has been successfully used to explain a range of puzzles in economics, such as the disposition effect, asymmetric price elasticity, elasticity of labor supply that are inconsistent with standard models of labor supply, and the excess sensitivity of consumption to income [2]. Expected utility theory and prospect theory have three main differences [4]. First, expected utility theory evaluates utility from the final states of wealth, which includes wealth from the prospect and other existing assets, whereas prospect theory evaluates the value of a prospect from a change in wealth due to a prospect. Second, expected utility theory uses stated probabilities to find the expected utility, where expected utility is the summation of utilities from each possible outcome, weighted by the probability of occurrence for each potential outcome. Prospect theory, however, uses decision weights in its value function. The decision weighting function proposed by Kahneman and Tversky [12] has the decision weights lower than the states' probabilities, except for extreme outcomes. Extremely low probability events beyond a certain benchmark are assigned a zero probability of occurrence, whereas those with an extremely high probability of occurrence are treated with certainty. Third, utility theory assumes that decision makers are risk aversion, risk neutral or risk seeking, but the same person cannot simultaneously exhibit risk aversion, risk neutrality and risk-seeking characteristics. On the other hand, prospect theory predicts that investors would be risk averse in gains and risk seeking in losses, regardless of their level of wealth.

Obviously, the decision-making based on prospect theory is more in line with people's actual decision-making behavior, and it is an important research topic about how to use prospect theory to multiple attribute decision making, because the models and methods of multiple attribute decision making are mainly based on expected utility theory. At present, the research on the risk MADM problems based on prospect theory is less. The main studies are shown as follows: Wang et al. [27] proposed a fuzzy MADM method with the prospect theory based on the multiple criteria decision making problems in which the criteria weight was unknown and the criteria values of the alternative took the form of the trapezoidal fuzzy numbers. In this method, the risk psychological factors of the decision-makers were introduced to the multiple criteria decision making, and the prospect value function of the trapezoidal fuzzy number was defined according to the prospect theory and the distance formula of the fuzzy numbers, and based on these, firstly, the nonlinear programming model of the maximize comprehensive prospect value of the alternative was constructed, then the optimum weight vector was obtained by solving this model, finally, the alternatives were sorted. In this paper, based on the idea of the prospect theory, the criteria weight was transformed into the weight function of the prospect theory, but the decision making problem itself was not the risk decision making problem. Hu and Zhou [9] proposed the multiple criteria decision making method for the risk decision making problem based on prospect theory. In this method, firstly, the multiple criteria decision making problems were described based on the prospect theory and the decision-making reference points of the criteria are determined, then the prospect values of the criteria for every alternative were calculated through the value function and the weight function, and the prospect values of the alternatives were calculated through the weighted average of them, finally, the alternatives were ranked based on the prospect values. But this method was good for solving these risk decision making problems in which the criteria values was the real number and the probability was the precise number, and it can not solve these risk decision making problems with the interval probability, in which the criteria values were the uncertain linguistic variables. Hu et al. [8] proposed a multiple criteria decision making method for the risk decision making problems with the linguistic evaluation information based on prospect theory. In this method, firstly, the linguistic information was transformed into the interval numbers in the decision making matrix, and the difference function between the interval numbers was defined and the reference points of the criteria were determined, and the prospect result matrix was calculated, then the prospect values of the criteria for every alternative were calculated through the value function and the weight function, and the prospect values of the alternatives can be obtained through the weighted average of them, finally, the