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# Symbolic and numerical analysis of plates in bending using Matlab ☆

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ABSTRACT

The implementation of the meshless collocation method using radial basis functions for solving partial differential equations involves the symbolic manipulation of governing equations. In the present work, the meshless method is used to solve the bending of plates, using a third order shear deformation theory. To avoid manually writing all the terms, the Symbolic Math Toolbox in Matlab is used to symbolically manipulate expressions. The final terms are written as Matlab functions to be used by the numerical main program.



## 1. Introduction

The use of symbolic–numeric programs is not a novelty in structural mechanics. Symbolic computation has been used mainly with finite element methods, for fast generation of stiffness matrices and to reduce numerical instabilities (Gunderson and Cetiner, 1971; Pedersen and Megahed, 1975; Pedersen, 1977; Kikuchi, 1989; Eisenberger, 1990; Wang, 1986). An extensive review on the use of symbolic computation in structural engineering can be found in Pavlovic (2003).

The numerical modeling of plates in bending and in structural mechanics in general is traditionally made using the finite element methods. This class of methods uses a mesh, which can cause problems in the case of discontinuities, strong singularities and complex geometries. In order to avoid mesh generation, meshless numerical methods are being increasingly used. Meshless methods do not use a mesh and are easier to implement. However, in the case of plates in bending, equations can be very long and tedious to write, leading to unnecessary typo mistakes by the user. As an example, for the

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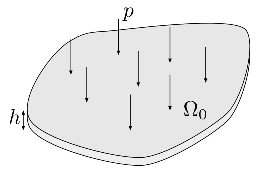


**Fig. 1.** Plate with thickness $h$.

third order shear deformation theory, five equations have to be considered in the domain, summing 257 terms to be written by the programmer.

In order to analyze the bending of plates using a meshless method but avoiding the extensive task of typing all expressions, two Matlab programs are developed. The first, writes automatically expressions for the equilibrium equations. The second caries out the necessary numerical calculations in order to run the numerical meshless method.

To the author's knowledge, there are no published works concerning the use of symbolic computation with Matlab applied to structural mechanics using meshless methods.

## 2. Problem

Composite plates are one of the most significant applications of composite materials in industry. Layers are stacked together to form thin or thick laminates. The problem of a composite plate in bending is considered. The plate with thickness $h$ and area $\Omega_0$ is subjected to an applied load $q(x, y)$ (Fig. 1).

The equilibrium equations for the problem of a plate in bending can be found by using the principle of virtual displacements:

$$\int_{\Omega_0} (\sigma_{ij}\delta\epsilon_{ij} - f_i\delta u_i)\,dv + \int_{\Gamma_\sigma} t_i\delta u_i\,ds = 0 \tag{1}$$

where $\sigma_{ij}$ is the stress tensor, $\delta\epsilon_{ij}$ are virtual strains, $f$ are body forces, $\delta u_i$ are virtual displacements, $\Omega_0$ is the volume of the undeformed body, $dv$ and $ds$ are the volume and surface elements of $\Omega_0$ and $t$ represents surface tractions.

Strains relate to a chosen displacement field, which makes assumptions on how the plate bends. In the present case, the third order shear deformation theory is used. The displacement field depends on five variables, $u(x, y)$, $v(x, y)$, $w(x, y)$, $\phi_x(x, y)$ and $\phi_y(x, y)$, giving origin to a set of five equations and five boundary conditions (see Section 3 for details). As an example, a simplified version of the first equation is presented, written in terms of displacements (2). The presented equation considers some simplifications such as symmetry of the stiffness matrix.

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2} + A_{12}\frac{\partial^2 v}{\partial y\partial x} + B_{11}\frac{\partial^2 \phi_x}{\partial x^2} + B_{12}\frac{\partial^2 \phi_y}{\partial y\partial x} - \frac{4}{3h^2}E_{11}\left(\frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right)\\
&\quad - \frac{4}{3h^2}E_{12}\left(\frac{\partial^2 \phi_y}{\partial y\partial x} + \frac{\partial^3 w}{\partial y^2\partial x}\right) + A_{33}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y\partial x}\right) + B_{33}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial y\partial x}\right)\\
&\quad - \frac{4}{3h^2}E_{33}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial y\partial x} + 2\frac{\partial^3 w}{\partial y^2\partial x}\right) = 0
\end{aligned} \tag{2}$$

All five equations present a similar structure: constant $\frac{\partial^n u}{\partial x^n}$ + constant $\frac{\partial^n u}{\partial y^n} + \cdots$.

The main objective is to solve the set of partial differential equations for variables $u(x, y)$, $v(x, y)$, $w(x, y)$, $\phi_x(x, y)$ and $\phi_y(x, y)$, using a meshless numerical method with radial basis functions (RBF). Terms $A_{ij}$, $B_{ij}$, $E_{ij}$ and $h$ are known constants.

In order to apply the RBF numerical method to solve the structural problem of a plate in bending, equations have to be written, in the present case, in Matlab. In the present paper, the symbolic