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On a hybrid analytical–experimental technique to assess the storage modulus of resilient materials using symbolic computation

H. Policarpo, M.M. Neves, N.M.M. Maia

IDMEC, Instituto Superior Técnico, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

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ABSTRACT

This paper presents details of symbolic computation used to develop hybrid analytical–experimental methods. These methods are considered rigorous since they are based on the exact analytical solution and not in an approximate discrete approach solution. This is illustrated through a simple example of solid mechanics, involving a boundary value problem with hyperbolic differential equation and Neumann boundary condition from the classical theory of longitudinal vibration of rods, in which the authors recur to symbolic computation (as revealed to be essential) to obtain a novel analytical expression and its solution. Then, by comparing the analytical and experimental responses, the unknown material parameters of the specimen are assessed through an inverse problem. The main idea may be used in different types of experiments. Here, a simple application from the longitudinal vibration of an elastic bar is presented for illustration purposes. It consists of a simple technique that has been developed by the authors to assess the storage modulus of resilient materials, as for e.g. composition cork-like materials. Compared to alternative procedures that use a numerical finite element method, the proposed method is much simpler; and while compared to the discrete two degrees-of-freedom model currently used in many laboratories it shows better accuracy.

1. Introduction

In science, theoretical concepts are used to model natural phenomena, through mathematical expressions containing numbers, variables, operators and functions. Those expressions are transformed

E-mail address: maneves@dem.ist.utl.pt (M.M. Neves).

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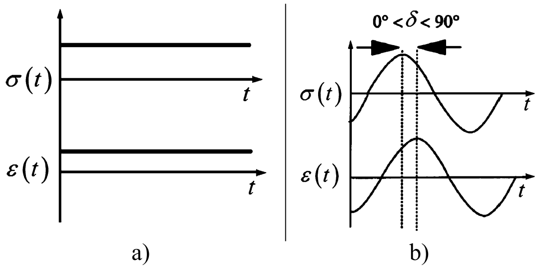

Fig. 1. Stress and strain versus time plots due to a: a) static load; b) dynamic harmonic load.

into "simpler" ones, for easier understanding of the phenomena, recurring to mathematical reasoning methods, see Cohen (2002, 2003).

Nowadays, with the use of computer programs it is possible to routinely simplify algebraic expressions (see e.g., D'Alfonso et al., 2011), integrate complicated functions (see e.g., Bailey and Borwein, 2011), find exact solutions to differential equations (see e.g., Poole and Hereman, 2011), and perform many other operations such as quantifier elimination (see e.g., Hong and Safey El Din, 2012) encountered in applied mathematics, science, and engineering.

Nonetheless, a considerable decline in the classical analytical methodologies has been verified mainly due to the remarkable development of numerical tools, like for e.g., the finite element (FE) method, and to computational power. Even though these numerical tools allow the analysis of complex problems, the existence of closed form solutions or the possibility to derive new ones by analytical means should not be discarded. Indeed, either one may often provide a (new) starting point that may (re)direct subsequent numerical runs when necessary, see Pavlović (2003). Furthermore, the analytical approach may provide a more elemental understanding of the significant physical occurrence, allowing us to attain additional sensitivity for the problem at hands, see Lee (2009).

In most cases, to model the dynamic behavior of mechanical systems, a discrete model consisting of mass, stiffness and damping discrete elements properly combined may be sufficient, see Pritz (1980). Alternatively, and usually for more complex systems, the FE method and analysis can be applied to obtain a good approach. Nevertheless, if the analytical solution can be derived or is available, which occurs in few systems, the exact dynamic behavior of mechanical systems can be obtained.

However, there is a wide range of materials used for mechanical systems, with considerably different behavior and thus, mechanical properties. In addition, the same material can present different behaviors (e.g., elastic, viscoelastic, elasto-plastic, etc.) depending on the types of loads (e.g., static, dynamic, etc.) and environmental conditions (e.g., temperature, humidity, etc.), among others.

Usually, Hooke's law, which states that the force F needed to extend or compress a spring by some displacement u is proportional by a factor k to that elongation (variation of length of the spring), i.e., $F = ku$, is also a first order linear approximation to the real response of other elastic bodies as long as the forces and deformations are small enough.

Thus, for a linear elastic bar with uniform cross sectional area A subjected to an uniaxial (e.g., along the x direction) traction or compression load F, Hooke's law relates the stress $\sigma = F/A$ in the bar to the strain $\varepsilon = \partial u/\partial x$ through $\sigma = E\varepsilon$, where E is the proportional constant usually designated as the modulus of elasticity or Young's modulus.

As previously stated, the type of loading may dictate different behaviors for the same material. In Fig. 1 examples of stress and response strain curves are illustrated for static and dynamic harmonic loads.

In the case of harmonic excitation with stress $\sigma(t) = \sigma_0 e^{\mathrm{i}\omega_{ap} t}$ and strain $\varepsilon(t) = \varepsilon_0 e^{\mathrm{i}(\omega_{ap} t - \delta)}$ – where σ_0 and ε_0 are the stress and strain amplitudes, respectively, $\mathrm{i} = \sqrt{-1}$ is the imaginary unit, ω_{ap} is the angular frequency of the applied force, t is the time and δ is the phase lag between the stress and the strain, as illustrated by Fig. 1 b) – the elasticity modulus must therefore be redefined using complex quantities to include the time dependency.

Hence, Hooke's law may be expressed at each material point as

$$\sigma(t) = E^* \varepsilon(t) \tag{1}$$