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Neglected critical issues of effective CAS utilization ☆

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ABSTRACT

This paper examines two neglected critical issues of the effective utilization of Computer Algebra System (CAS). By using a number of examples from an upper secondary mathematics education, these issues deal with the instrumentalization of CAS, and support to students when solving tasks by CAS (i.e. CAS-based task design). Regarding this instrumentalization, the paper considers the extent to which it can be done with CAS tools at present, and discusses several critical issues in doing so with respect to tool, task and designer (learner). Concerning this support, the paper calls for a detailed task design that also includes the issues of acceptable solutions and scaffolding offered, which should be, in some detail, clarified and given in examination materials. Several challenges relating to the two critical issues are considered.

1. Introduction

As given in its title, this paper deals with the problem of using CAS effectively. This is an important problem bearing in mind the Vigotskian view that the use of tools improves our thinking. (This view probably originated in the Renaissance: *Nec manus, nisi intellectus, sibi permissus, multum valent*: *instrumentis et auxiliis res perficitur*. – Francis Bacon.) Although there is a framework that describes progress in CAS use and gives several critical issues in using CAS effectively (Pierce and Stacey, 2004), two

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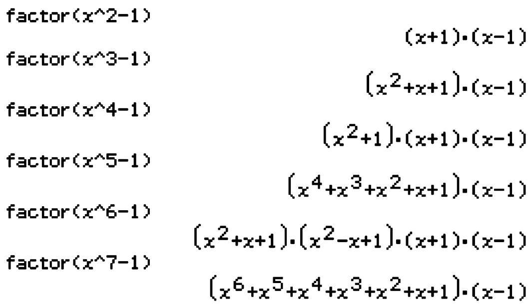

Screenshot 1. Factorizations of $x^n - 1$ for six values of n by Casio ClassPad Manager.

important critical issues have been neglected by CAS-based research. Let us briefly introduce these issues and then present the content of this paper and its relevance to further development of the field.

Computer Algebra System (CAS) usually solves problems in ways that are not open to its users. For example, its user does not know whether the roots 2 and 3 of the equation $x^2 - 5x + 6 = 0$ are found by the application of the well-known formula for the roots of the quadratic equation, or by using the following factorization $x^2 - 5x + 6 = (x - 2)(x - 3)$. Also, techniques used by CAS typically do not reflect techniques used in paper-and-pencil work. The student traditionally factors $x^n - 1$ as $(x-1)(x^{n-1} + x^{n-2} + \cdots + 1)$, where, as presented in Screenshot 1, CAS may do this in a puzzling way, even changing the assumed order of the factors (example suggested by Kieran and Saldanha, 2008).

Research evidences that these two unfavorable CAS features (the lack of transparency and congruence) may constrain students' success in learning mathematics with CAS to a large extent (e.g., Drijvers, 2004). Such a negative impact can be reduced with appropriately designed user-defined commands (Gjone, 2009; Kadijevich, 2009). By writing and using such commands the user in fact enriches CAS. Such CAS enrichment is attained through a process called *instrumentalization*. Through a complementing process called *instrumentation* CAS built-in affordances shape the user's thinking and actions, and these two processes constitute the so-called instrumental genesis through which the user gradually builds an instrument from an artifact (Trouche, 2005). Although researchers refer to the process of instrumentalization (e.g., Fuglestad et al., 2010), they have not examined it in detail, focusing on, for example, main difficulties in programming the tool.

An appropriate instrumentalization, resulting in an improved CAS that better suits the users' needs, is one important critical issue in using CAS effectively. Let us mention the other critical issue. Because the ability of CAS is still limited (e.g. no CAS can verify equivalence of expressions in all cases as the problem of equivalence of expressions is simply not decidable; Richardson, 1968), it is vital to provide some support to students who solve tasks by CAS, which, for example, indicate when to use its affordances with care. For example, CAS usually simplifies $\frac{x^2}{x}$ to x, not adding the constraint $x \neq 0$. This support should also clarify what solutions to tasks given are acceptable. Can, for example, the student use visual arguments, and if so, in what way? Although advanced technology has been used in solving mathematical tasks since the end of 20th century, research is only beginning to examine the features of computer-based mathematical tasks (Berger, 2011). Among these features, the issues of required solution and scaffolding offered are still missing.

The following two parts, Parts 2 and 3, examine the issues of intrumentalization and support (i.e. CAS-based task design) in detail. While Part 2 examines to what extent CAS may be instrumentalized at present and which issues may be critical in doing so, Part 3 discusses what support in solving tasks by CAS should be given to students and indicates issues that are not easy to resolve. The originality of this paper should be found in the treatment of the issues of CAS intrumentalization and CAS-based task design, which despite their high relevance to effective CAS utilization, have been neglected so far by CAS-related research. The contribution of the paper is non-trivial because it not only enables teachers to improve the use of CAS considerably, but also gives important directions for