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Requiem for the max rule?

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A B S T R A C T

In tasks such as visual search and change detection, a key question is how observers integrate noisy measurements from multiple locations to make a decision. Decision rules proposed to model this process have fallen into two categories: Bayes-optimal (ideal observer) rules and ad-hoc rules. Among the latter, the maximum-of-outputs (max) rule has been the most prominent. Reviewing recent work and performing new model comparisons across a range of paradigms, we find that in all cases except for one, the optimal rule describes human data as well as or better than every max rule either previously proposed or newly introduced here. This casts doubt on the utility of the max rule for understanding perceptual decision-making.

1. Introduction

Since the dawn of psychophysics, its ambition has been to reveal the workings of the brain's information-processing machinery by only measuring input–output characteristics. This ambition is normally pursued by conceptualizing the transformation from input to output as a concatenation of an encoding stage, in which the sensory input is internally represented in a noisy fashion, and a decision stage, in which this internal representation is mapped to task-relevant output. In the simplest of models of the simplest of tasks, the internal representation is modeled as a scalar measurement and the decision stage as the application of a criterion to this measurement. Unfortunately, this most basic form of signal detection theory has limited mileage when it comes to bridging the gap between laboratory and real-world tasks. One reason for this is that real-world decisions often involve integrating information from multiple locations – looking for a person in a crowd, detecting an anomaly in an image, or judging a traffic scene. In the laboratory, the essence of such tasks can be mimicked by presenting multiple stimuli and asking for a "global" judgment, i.e. one which requires the observer to take all stimuli into consideration. In such tasks, even if the internal representation of an individual stimulus is modeled as a scalar measurement, the internal representation of the entire stimulus array is a vector, and the decision stage consists of mapping this vector to task-relevant output.

At least for the past sixty years, in multiple-item tasks requiring a global judgment, psychophysicists have been searching for mappings of this kind that are both mathematically cogent and adequately describe human behavior. Rules that have been proposed have mostly come in two types: optimal rules and simple ad-hoc rules. According to optimal (or Bayes-optimal, or ideal-observer, or likelihood ratio) rules (Green & Swets, 1966; Peterson, Birdsall, & Fox, 1954), observers maximize a utility function by using knowledge of the statistical process that generated the internal representations. When the utility function is overall accuracy, as it often is assumed to be, optimal decision-making reduces to choosing the option that has the highest posterior probability given the current sensory observations (MAP estimation). The notion of an optimal decision rule is general: such a rule can be derived for any task, without having to make task-specific assumptions beyond the formalization of the experimental design.

There are, however, reasons to consider alternatives to optimal decision rules. First, these rules often take a complicated form, meaning that evaluating response probabilities under the optimal model was cumbersome for the digital computers available in the 1960s (Nolte & Jaarsma, 1967); this is much less of a consideration nowadays. Second, observers might not have knowledge of all the task statistics that are needed to compute the optimal rule, or neural implementation of that rule might be infeasible; these are still valid motivations for considering alternative decision rules.

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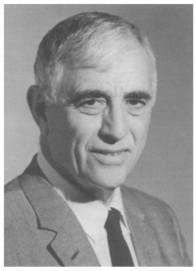

Bernard Osgood Koopman
(image from Morse, 1982)

Of all alternatives to optimal decision rules in multiple-item global judgment tasks, the most prominent might be the maximum-of-outputs rule, or max rule. This rule dates back to at least the French–American mathematician Bernard Koopman (Koopman, 1956; Morse, 1982), who considered the problem of making N glimpses to determine whether a signal is present, for example during underwater echo ranging. When time (glimpses) is translated to space (locations), this problem is equivalent to detecting whether a signal is either present at all N locations, or absent at all. Koopman assumed that the observer makes a decision on every glimpse, and makes an overall decision using an "or" operation, which means that the observer reports "present" if any of the individual decisions returns "present". Assuming that every individual "present" decision is made when an underlying continuous decision variable exceeds one specific criterion, Koopman's decision model is equivalent to one in which the observer decides that the signal is present if the largest of those decision variables among all locations exceeds that criterion – hence the terminology "max rule". Since Koopman, the max rule has been considered by many greats of signal detection theory (Graham, Kramer, & Yager, 1987; Green & Swets, 1966; Nolte & Jaarsma, 1967; Palmer, Verghese, & Pavel, 2000; Pelli, 1985; Shaw, 1980; Swensson & Judy, 1981), although predominantly in a different context, namely the problem of detecting one signal among N locations.

When the observer knows the statistics of the sensory observations used to make the decision, the max rule is not the best strategy either in the N-of-N problem Koopman considered or in his successors' one-of-N problem. Moreover, the max model will need to be modified in ad-hoc ways whenever the task is changes (we will encounter examples of this). Of course, in spite of this suboptimality and lack of generalizability, the max model might be a better description of human behavior than the optimal model in these or other tasks. In this paper, we will argue that this does not seem the case, and that the optimal rule provides an equally good or better account of the data than every max rule in almost every experiment examined.

A note on nomenclature might be helpful. In the classification scheme of (Ma, 2012), we distinguished the notions of Bayesian, optimal, and probabilistic decision rules in perception. Bayesian rules are based on posterior distributions, a rule that is optimal (in a "relative" sense) maximizes performance given sensory noise, and probabilistic rules take into account the quality of sensory evidence on a trial-to-trial basis. An observer can be Bayesian but not optimal, for example when they use previously learned priors rather than the ones appropriate for the experiment. According to this classification, the optimal rules we will consider are both Bayesian and probabilistic, whereas the max rules are suboptimal, non-Bayesian, and in most cases also non-probabilistic.

1.1. Scope

In this paper, we consider visual decision-making tasks that meet the following criteria.

(1) The observer briefly views either an array of N stimuli, or two arrays of N stimuli separated in space and/or time.
(2) The observer makes a single categorical judgment about these stimuli.
(3) The categories are defined in terms of a small number of easily parameterizable features.
(4) All stimuli are relevant to the category decision.
(5) Trials are independent.

We will call these tasks "feature-based global categorization tasks", although (5) is not captured by that term. This category class of tasks encompasses many common paradigms, such as:

- Visual search: One or more targets are drawn from a target distribution, and the remaining items are drawn from a distractor distribution. Common sub paradigms include:
  - Detecting the presence of one or more targets among distractors.
    - Perhaps the most studied task of this type involves a single target that takes on one fixed value, and distractors that are independently drawn from a distractor distribution.
    - Oddity detection: there is a single target whose value varies from trial to trial, and the distractors are identical to each other (homogeneous distractors) but their common value is also variable.
    - Sameness judgment: on a target-present trial, all items are targets, and the targets are identical to each other; on a target-absent trial, the distractors are not identical to each other.
  - Localizing one or more targets that are present among distractors.
  - 2AFC on which of two arrays contained the target.
  - Categorizing one or more targets that are present among distractors.
    - Example 1: was the tilted bar among the vertical bars tilted left or right?
    - Example 2: all items are targets, the target orientations are drawn independently from the same Gaussian distribution, and the observer reports whether the mean of this distribution was tilted left or right.
- Change detection.
  - Detecting the occurrence of one or more changes.
  - Localizing one or more changes.
  - Categorizing one or more changes.

In this paper, we will not discuss experiments using natural scenes or real-world objects, ones in which only one stimulus is relevant for the decision (such as discrimination at a cued location), ones in which the stimuli are displayed until the subject makes a decision, ones involving crowding, and spatial integration tasks such as judging whether two orientations belong to the same contour (since those rely on categories that are defined not only in terms of the features of the stimuli, but also their spatial locations).