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How metamer mismatching decreases as the number of colour mechanisms increases with implications for colour and lightness constancy

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ABSTRACT

Metamer mismatching has been previously found to impose serious limitations on colour constancy. The extent of metamer mismatching is shown here to be considerably smaller for trichromats than for dichromats, and maximal for monochromats. The implications for achromatic colour perception are discussed.

1. Introduction

The colour of a reflecting object does not seem to alter much as the illumination changes. This phenomenon, known as colour constancy, poses serious problems for colour vision theory (Brainard & Radonjic, 2014; Foster, 2011). Colour constancy is usually understood as meaning that a change in illumination results in a transformation of the cone excitations induced by the light reflected by the object. A solution has been sought in the way enunciated by Helmholtz (1867) and elaborated upon by others (e.g., Ebner, 2007; Gijsenij, Gevers & Weijer, 2010); namely, to find an inverse transformation of the cone excitations. However, because of metamer mismatching (Wyszecki & Stiles, 1982), such an inverse transformation (accounting for the illumination change) cannot exist in principle (Logvinenko, 2013; Logvinenko, Funt & Godau, 2014). Indeed, two reflecting objects that invoke the same cone excitations under one light can produce different cone excitations under a second light (Wyszecki & Stiles, 1982). Hence, being an unavoidable obstacle to any inverse (compensating) transformation, metamer mismatching imposes certain limits on colour constancy. Quantitative analysis of the extent of metamer mismatching in terms of a *metamer mismatch index* showed that metamer mismatching is rather large even for CIE illuminants $D65$ and A (Logvinenko, Funt & Godau, 2014; Logvinenko et al., submitted for publication).

Interestingly, colour constancy has been reported not only for trichromats but for dichromats as well (Baraas et al., 2010; Ruttiger et al., 2001), with some of these researchers reporting that colour constancy of dichromats is poorer than that of trichromats (Baraas et al., 2010). In view of this it is of interest to compare metamer mismatching for dichromats and trichromats; and such a comparison is made here. For the sake of generality metamer mismatching is also evaluated for monochromatic vision. The results are presented below.

2. The extent of metamer mismatching for monochromatic, dichromatic and trichromatic vision

Given a point in the cone excitation space induced by some spectral reflectance under some illuminant (I_1), metamer mismatching under another illuminant (I_2) reveals itself in the *metamer mismatch volume*, which is the set of the cone excitations induced under illuminant I_2 by all the reflectances that are metameric to one another (i.e., map into the single point in the cone excitation space) under illuminant I_1. Fig. 1 presents the metamer mismatch volumes in the CIE 1931 colorimetric space for nine flat reflectances (i.e., of the form $x(\lambda) = k$ for nine values of the

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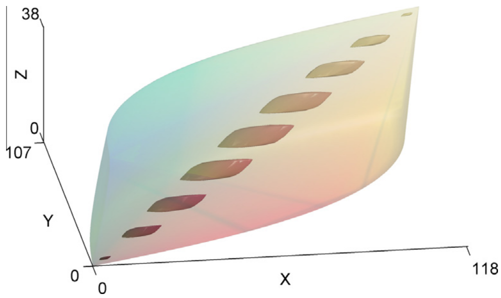

Fig. 1. Metamer mismatch volumes plotted inside the trichromatic object-colour solid produced for CIE illuminant *A*. The volumes are induced by a change in illuminant from CIE *D*65 to CIE *A* for nine flat reflectances mapping to points lying along the achromatic interval connecting the black and white poles of the object-colour solid.

constant k) mapping to the achromatic interval (i.e., the interval connecting the black and white points) induced by a transition from CIE illuminant *D*65 to CIE illuminant *A*.

Fig. 2 presents the metamer mismatch "volumes" for the dichromatic case obtained by using the CIE 1931 $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ colour matching functions for the same 9 reflectances. Of course, in this case the metamer mismatch volumes degenerate into areas. Notably, the metamer mismatch areas in Fig. 2 compared to the dichromatic object-colour solid area appear to be larger than the metamer mismatch volumes compared to the trichromatic object-colour solid volume in Fig. 1. Fig. 3 shows the metamer mismatch "volumes" (degenerating to intervals) for the monochromatic case obtained by using just the CIE 1931 $\bar{y}(\lambda)$ colour matching function for the same 9 reflectances.

To quantify the extent of metamer mismatching an index (referred to as the *metamer mismatch index*) has been defined as a ratio of the metamer mismatch volume to the volume of the object-colour solid[1] (Logvinenko & Levin, submitted for publication; Logvinenko, Funt & Godau, 2014). Metamer mismatch indices corresponding to these metamer mismatch volumes can be found in Figs. 4 and 5. As one can see, the extent of metamer mismatching essentially depends on the dimensionality of colour vision, progressively increasing from trichromatic vision through dichromatic to monochromatic. The amount of metamer mismatching in the monochromatic case is so large that it deserves special consideration. One more reason to pay extra attention to the monochromatic case is that it provides a good opportunity to look into the logic of the metamer mismatching calculation, which in the one-dimensional case is rather simple.[2]

3. Metamer mismatching for monochromatic vision

Consider a luminance channel (i.e., the CIE $\bar{y}(\lambda)$ colour matching function) and two illuminants: CIE *D*65 and *A* (with spectral power distributions $p_{D65}(\lambda)$ and $p_A(\lambda)$). The luminance of the light reflected by an object with spectral reflectance function $x(\lambda)$ under illuminant *D*65 is given by

$$L_{D65}(x) = \int_{\lambda_{\min}}^{\lambda_{\max}} x(\lambda)p_{D65}(\lambda)\bar{y}(\lambda)d\lambda, \quad (1)$$

where $[\lambda_{\min}, \lambda_{\max}]$ is the visible spectrum wavelength interval; and under illuminant *A* by

[1] The object-colour solid is the set of the cone excitations produced by all the spectral reflectance functions Schrödinger, 1920; Wyszecki and Stiles, 1982.

[2] The calculation of metamer mismatch volumes in the n-dimensional case with arbitrary integer n is described at length elsewhere (Logvinenko & Levin, submitted for publication; Logvinenko, Funt & Godau, 2014).

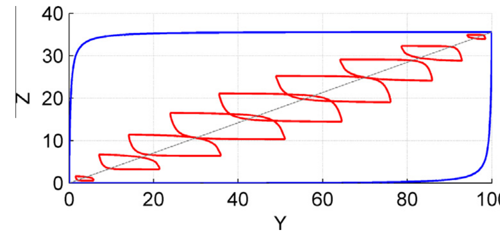

Fig. 2. Metamer mismatch "volumes" (i.e., areas) for the same nine flat reflectances as in Fig. 1 and mapping to points along the achromatic axis (light grey line) of the dichromatic object colour solid (blue contour) for CIE illuminant *A*. The boundaries of the mismatch areas are shown in red. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

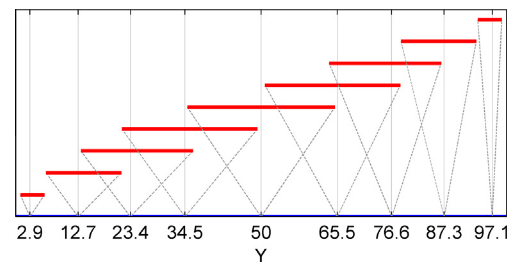

Fig. 3. Metamer mismatch "volumes" (i.e., intervals) for the nine flat reflectances (presented in Figs. 1 and 2) for a change in illuminant from CIE *D*65 to CIE *A* in the monochromatic case. The intervals are shown by horizontal red bars, which have been offset vertically for clarity of presentation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

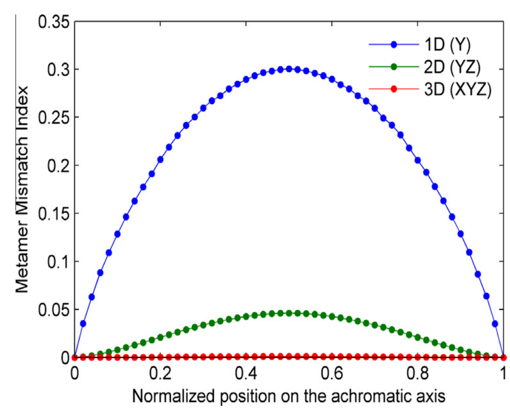

Fig. 4. Metamer mismatch indices for the trichromatic, dichromatic and monochromatic cases for points lying along the achromatic interval. The abscissa is the relative location along the achromatic interval from the black to white poles of the respective object-colour solid.

$$L_A(x) = \int_{\lambda_{\min}}^{\lambda_{\max}} x(\lambda)p_A(\lambda)\bar{y}(\lambda)d\lambda. \quad (2)$$

Taking Eqs. (1) and (2) as defining formally two abstract colour mechanisms ($L_{D65}(x)$ and $L_A(x)$), consider the object-colour solid for these two mechanisms (Fig. 6). It is an area in the colour mechanism output space[3] that encompasses the pairs of the luminance outputs ($L_{D65}(x)$ and $L_A(x)$) produced by all possible spectral reflectance functions $x(\lambda)$.

[3] That is, in a plane where the outputs of the luminance mechanisms serve as Cartesian coordinates.