Knowledge-Based Systems 100 (2016) 59–73

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Definite integrals of multiplicative intuitionistic fuzzy information in decision making

Shan Yu [a], Zeshui Xu [b,c,*]

[a] School of Sciences, PLA University of Science and Technology, Nanjing, Jiangsu 210007, China
[b] Business School, Sichuan University, Chengdu, Sichuan 610064, China
[c] Collaborative Innovation Center of Social Safety Science and Technology, Nanjing, Jiangsu, China

ARTICLE INFO

Article history:
Received 28 September 2015
Revised 1 February 2016
Accepted 8 February 2016
Available online 26 February 2016

Keywords:
Decision making
Multiplicative intuitionistic fuzzy function
Fundamental theorem of calculus
Subtraction definite integral
Division definite integral

ABSTRACT

In this paper, we investigate the definite integrals of multiplicative intuitionistic fuzzy information in decision making. Firstly, we propose a new order of multiplicative intuitionistic fuzzy numbers (MIFNs) and study its properties. Secondly, we consider the limiting process from which the definite integrals of multiplicative intuitionistic fuzzy functions (MIFFs) over a closed interval are derived. Furthermore, we study the forms of indefinite integrals, deduce the fundamental theorem of calculus, derive the concrete formulas for ease of calculating definite integrals from different angles, and discuss some useful properties of the proposed definite integrals. Finally, we give applications of the subtraction definite integral of MIFFs in decision making.

1. Introduction

To describe the input information more comprehensively, Atanassov [1] proposed the notion of intuitionistic fuzzy set (A-IFS), which uses the symmetrical scale and adds a non-membership function and a hesitancy function to each element of the fuzzy set [2]. Since its introduction, the A-IFS theory has been investigated deeply and widely applied in various fields of modern society [3–7], especially its wonderful applications in decision making.

As the most common techniques in decision making, two main forms of preference relations: fuzzy preference relations [8] and multiplicative preference relations [9] have been developed. A fuzzy preference relation (see Table 1.1 for more details) uses the 0.1–0.9 scale, which assumes that the grades between "Extremely not preferred" and "Extremely preferred" are distributed uniformly and symmetrically, while on the other hand, as stated in Refs. [10–13], there exist the problems that need to assess their variables with the grades which are non-uniformly and unsymmetrically distributed owing to the complexity of the objective world. In this situation, the 1–9 scale proposed by Saaty [9] demonstrates its contribution especially in expressing a multiplicative preference relation S (see Table 1.1 for more details). Since the basic element s_{ij} in S is not symmetrically distributed around 1, then it can express the decision makers' preferences more objectively in the situations discussed above.

Due to the complexity and uncertainty involved and incomplete information or knowledge gained in the real-life decision making problems, it is difficult to provide a precise preference relation. In these circumstances, the corresponding interval-valued multiplicative fuzzy preference relation may be given (see Table 1.1 for more details). Since an interval-valued fuzzy number $\tilde{b}_{ij} = [b_{ij}^-, b_{ij}^+]$ is mathematically equivalent to an intuitionistic fuzzy number (IFN) $\beta_{ij} = (\mu_{ij}, \nu_{ij}) = (b_{ij}^-, 1 - b_{ij}^+)$, then the interval fuzzy preference relation $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ is mathematically equivalent to the intuitionistic fuzzy preference relation $B = (\beta_{ij})_{n\times n}$ [14], and both of them satisfy the following conditions:

$$\mu_{ij} = \nu_{ji},\ \nu_{ij} = \mu_{ji},\ 0 \le \mu_{ij} + \nu_{ij} \le 1 \text{ and } 0 \le \mu_{ij}, \nu_{ij} \le 1$$

Furthermore, it is important to mention that for containing not only the membership function but also the non-membership function, the IFNs can describe the preference relation more comprehensively than the interval-valued fuzzy numbers. Similarly, motivated by the interval-valued multiplicative preference relation (see Table 1.1 for more details), Xia et al. [15] proposed the multiplicative intuitionistic fuzzy number (MIFN) $\alpha_{ij} = (\rho_{ij}, \sigma_{ij}) = (\alpha_{ij}^-, 1/\alpha_{ij}^+)$ and the multiplicative intuitionistic fuzzy preference relation $A = (\alpha_{ij})_{n\times n}$, satisfying:

$$\rho_{ij} = \sigma_{ji},\ \sigma_{ij} = \rho_{ji},\ 0 < \rho_{ij}\sigma_{ij} \le 1 \text{ and } 1/9 \le \rho_{ij}, \sigma_{ij} \le 9$$

* Corresponding author at: Business School, Sichuan University, Chengdu, Sichuan 610064, China. Tel.: +86 2584483382.
E-mail addresses: shan-shan-yu@163.com (S. Yu), xuzeshui@263.net (Z. Xu).

http://dx.doi.org/10.1016/j.knosys.2016.02.007
0950-7051/

Table 1.1
Two main kinds of preference relations and their extended forms.

	Based on fuzzy preference relation	Based on multiplicative preference relation
Traditional	$R=(r_{ij})_{n\times n},\ r_{ij}+r_{ji}=1,\ 0\le r_{ij}\le 1$ Using the 0.1–0.9 scale (uniformly and symmetrically)	$S=(s_{ij})_{n\times n},\ s_{ij}\cdot s_{ji}=1,\ 1/9\le s_{ij}\le 9$ Using Saaty's 1–9 scale (non-uniformly and unsymmetrically)
Interval-valued	$\tilde{B}=(\tilde{b}_{ij})_{n\times n},\ \tilde{b}_{ij}=[b_{ij}^-,b_{ij}^+]$, $b_{ij}^+ + b_{ji}^- = b_{ij}^- + b_{ji}^+ = 1$ $0\le b_{ij}^-\le b_{ij}^+\le 1$	$\tilde{A}=(\tilde{\alpha}_{ij})_{n\times n},\ \tilde{\alpha}_{ij}=[\alpha_{ij}^-,\alpha_{ij}^+]$, $\alpha_{ij}^+\cdot\alpha_{ji}^- = \alpha_{ij}^-\cdot\alpha_{ji}^+ = 1$ $1/9\le\alpha_{ij}^-\le\alpha_{ij}^+\le 9$
Intuitionistic fuzzy	$B=(\beta_{ij})_{n\times n}=((\mu_{ij},\nu_{ij}))_{n\times n}=((b_{ij}^-,1-b_{ij}^+))_{n\times n}$ $\mu_{ij}=\nu_{ji},\ \nu_{ij}=\mu_{ji}$ $0\le\mu_{ij}+\nu_{ij}\le 1,\ 0\le\mu_{ij},\nu_{ij}\le 1$	$A=(\alpha_{ij})_{n\times n}=((\rho_{ij},\sigma_{ij}))_{n\times n}=((\alpha_{ij}^-,1/\alpha_{ij}^+))_{n\times n}$ $\rho_{ij}=\sigma_{ji},\ \sigma_{ij}=\rho_{ji}$ $0<\rho_{ij}\cdot\sigma_{ij}\le 1,\ 1/9\le\rho_{ij},\sigma_{ij}\le 9$

It is vital to note that α_{ij} not only is superior to $\tilde{\alpha}_{ij}=[\alpha_{ij}^-,\alpha_{ij}^+]$ for its composition of two parts, but also makes foundation on the unbalanced scale, Saaty's 1–9 scale, which has many advantages in some circumstances [16–20].

Motivated by the above discussions, in this paper, we pay attention to the continuous distributed multiplicative intuitionistic fuzzy information [21], i.e., triangle, having coordinates of its vertices $(1/9,9)$, $(1/9,1/9)$, $(9,1/9)$, and a curve $\rho\sigma=1$, which fully inherits the operational laws defined in the multiplicative intuitionistic fuzzy sets (MIFSs) [15], and then we discuss its analytical nature in mathematical ways.

In Table 1.1, both γ_{ij} and s_{ij} indicate the degrees that the alternative x_i is preferred to x_j respectively. Both $\tilde{b}_{ij}$ and $\tilde{\alpha}_{ij}$ denote the interval-valued preference degrees or the intensities of the alternative x_i is preferred to x_j respectively. For $\beta_{ij}=(\mu_{ij},\nu_{ij})$, μ_{ij} indicates the degree that the alternative x_i is preferred to x_j, and ν_{ij} indicates the degree that the alternative x_i is not preferred to x_j, so it does with $\alpha_{ij}=(\rho_{ij},\sigma_{ij})$ in A.

On the other hand, as we know, information fusion is an important part in decision making. Since the data information collected may not be independent but associated, as well as their weight vector would also depend on the support level from the others, in this circumstance, other prepotent ways to aggregate correlative data must be found. Among the existing fusion methods, which have considered a certain degree of relationship between the data, integration plays a very important role, including Choquet integral [22], Sugeno integral [23], Riemann integral [24], and integrals on intuitionistic fuzzy set [25], etc. Based on them, several valid operators have been proposed, such as the induced Choquet integral (I-COA) operator [26], the generalized Bonferroni mean operators [27], Choquet integrals of weighted intuitionistic fuzzy information [28], intuitionistic fuzzy-valued fuzzy measures and integrals [29] and so on. Specially, Tan and Chen [30] pointed out that "intuitionistic fuzzy Choquet integral is extended from the IFOWA operator and the IFWA operator". Yu [31] interpreted that the Archimedean operation based interval-valued multiplicative intuitionistic fuzzy Choquet weighted averaging (geometric) operators can deal with the situations where the aggregated arguments are correlated. However, the existing integration theory based on the proposed measures cannot deal with continuous information, that is a relatively not pretty fulfilling in comparison with the infinitesimal calculus of the real number field. Motivated by this, Lei et al. [32] proposed two novel subtraction and division operations of IFNs, and developed a sequence of general integrals dealing with continuous intuitionistic fuzzy data based on Archimedean t-conorm and t-norm, whose special cases are the additive and multiplicative definite integrals defined in Ref. [33]. Are there any aggregation operators which can not only inherit the advantages of the Choquet integral aggregation technique, i.e., taking the importance of each discrete element into consideration and reflecting their relationships, but also aggregate the continuous multiplicative intuitionistic fuzzy information? As far as we know that nothing has been done about the aggregation of correlative or continuous multiplicative intuitionistic fuzzy information up to now. In this paper, we will propose a new family of aggregation operators based on the subtraction definite integral of multiplicative intuitionistic fuzzy functions (MIFFs) in order to get the correlative and continuous multiplicative intuitionistic fuzzy information fused. Such aggregation operators provide an excellent complement to the results of Refs. [32,33].

From the above analysis, in view of the defined operational laws in MIFSs and some results of our previous work [21,34], in this paper, we attempt to utilize multiplicative intuitionistic fuzzy theory and the calculus theory in real number field to investigate the general integral models (named as definite integrals) of the fusion methods for multiplicative intuitionistic fuzzy information, deduce the concrete mathematical formulas of the integration models to restore some commonly used aggregation operators, and apply their useful properties to aggregate the correlative or continuous multiplicative intuitionistic fuzzy information. The remainder of this paper is organized as follows: In Section 2, we review some necessary knowledge and the foregoing work related to the integrations and differentials of MIFFs. In Section 3, we define a new order of multiplicative intuitionistic fuzzy numbers (MIFNs), and get some basic properties for ease of the coming discussion. Then we pay attention to the subtraction definite integral in Section 4. Similar to the integral theory of Riemann in the real number field, we get the formal definition of the subtraction definite integral by three steps, that is, partition, sum and limit. Then we develop a mathematical formula to calculate the subtraction definite integral of MIFFs exactly. In order to show the simple way of calculating the subtraction integral more clearly, we give a simple example correspondingly. In Section 5, we discuss some useful properties of the subtraction definite integral. In Section 6, we get some similar results for the division definite integral of MIFFs, and in Section 7, we give the possible applications of the subtraction definite integral of MIFFs in decision making to make the paper more understandable. Finally, we conclude the paper in Section 8.

2. Preliminaries

In what follows, we introduce some basic concepts and operations, which are useful in the next sections:

Definition 2.1 [15]. Let X be a fixed set, a multiplicative intuitionistic fuzzy set (MIFS) is defined as:

$$A=\{(x,\langle\rho_A(x),\sigma_A(x)\rangle)|x\in X\}$$

which assigns to each element x a membership value $\rho_A(x)$ and a non-membership value $\sigma_A(x)$, with the conditions:

$$1/9\le\rho_A(x),\sigma_A(x)\le 9,\ \ \rho_A(x)\sigma_A(x)\le 1,\ \ x\in X$$

Specially, the ordered pair $\alpha=(\rho_\alpha,\sigma_\alpha)$ is called a multiplicative intuitionistic fuzzy number (MIFN), which satisfies the