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Age-dependent Fourier model of the shape of the isolated *ex vivo* human crystalline lens ☆

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ABSTRACT

Purpose: To develop an age-dependent mathematical model of the zero-order shape of the isolated *ex vivo* human crystalline lens, using one mathematical function, that can be subsequently used to facilitate the development of other models for specific purposes such as optical modeling and analytical and numerical modeling of the lens.
Methods: Profiles of whole isolated human lenses ($n = 30$) aged 20–69, were measured from shadow-photogrammetric images. The profiles were fit to a 10th-order Fourier series consisting of cosine functions in polar-co-ordinate system that included terms for tilt and decentration. The profiles were corrected using these terms and processed in two ways. In the first, each lens was fit to a 10th-order Fourier series to obtain thickness and diameter, while in the second, all lenses were simultaneously fit to a Fourier series equation that explicitly include linear terms for age to develop an age-dependent mathematical model for the whole lens shape.
Results: Thickness and diameter obtained from Fourier series fits exhibited high correlation with manual measurements made from shadow-photogrammetric images. The root-mean-squared-error of the age-dependent fit was 205 μm. The age-dependent equations provide a reliable lens model for ages 20–60 years.
Conclusion: The contour of the whole human crystalline lens can be modeled with a Fourier series. Shape obtained from the age-dependent model described in this paper can be used to facilitate the development of other models for specific purposes such as optical modeling and analytical and numerical modeling of the lens.

1. Introduction

There is much interest in computational modeling for the analysis of accommodative functions of the human crystalline lens. For such modeling to be valid, the geometrical, optical and mechanical parameters of the accommodative system need to be established. In the first instance, as it sets the initial conditions, a reliable geometric model of the lens will, to a large extent, impact the validity of data obtainable from computational models. Geometrically, the crystalline lens is a solid in three-dimensional space that approximately possesses one axis of rotational symmetry. The presence of rotational symmetry provides for simplification by modeling in axi-symmetric two-dimensions. However, the presence of anterior and posterior surfaces, as well as regions of vertical surfaces at the equator, invokes certain geometric conditions in modeling.

Chien, Huang, and Schachar (2003) listed five compulsory requirements for any analytical function that seeks to represents the shape of the crystalline lens to meet geometric conditions. The five compulsory requirements are: (1) the lens profile should be continuous and smooth, (2) the derivative at the pole should be zero, (3) the lens profile should be zero at the equator, (4) the slope at the equator should be vertical and (5) the surface slope should decrease monotonically as distance from the axis increases so that the generated surface has a positive Gaussian curvature everywhere.

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They also listed four desirable conditions, for the surface to have appropriate optical qualities, which are: (1) the model should follow the original lens profile closely, (2) the radii of curvature should be continuous, (3) the rate of change of the radii of curvature should be zero at the poles and (4) The rate of change of the radius of curvature with respect to distance from the axis should be gentle at least in the pole regions.

Against the foregoing geometric conditions, the majority of existing lens models describe the lens with two mathematical functions, one each for the two surfaces of the lens (Borja et al., 2008; Chien et al., 2003; Dubbelman & van der Heijde, 2001; Howcroft & Parker, 1977; Koretz, Handelman, & Brown, 1984; Manns et al., 2004; Rosen et al., 2006; Strenk, Strenk, Semmlow, & DeMarco, 2004). These models were mostly developed for supporting optical modeling and therefore focus on the central 4–5 mm region, bypassing the need for accurate portrayal of the lens equator. As an alternative to using two functions to describe anterior and posterior lens surfaces, Kasprzak (2000) and Smith, Atchison, Iskander, Jones, and Pope (2009) approximated the whole profile of the human lens using hyperbolic cosine functions and a generalized conic function.

It would be ideal to develop one mathematical model for the lens shape that can be applied for various purposes such as optical and mechanical modeling. But this would involve imposing various constraints to the lens shape and its subsequent derivatives, which would prohibit the model from being faithful to the true lens shape. A faithful age-dependent mathematical model of the zero-order shape of the lens contour would be beneficial to scientists who wish to develop their own models with constraints for specific purposes. This model would serve as a substitute for actual lens measurements.

We recently developed two age-dependent polynomial models (Urs et al., 2009). The Two Curves Model (TCM) used two 10th-order even polynomials to describe the two lens surfaces from equator to equator and the One Curve Model (OCM) described one meridional half of the lens using one 10th-order polynomial equation from pole to pole. While both models were age-dependent and attempted to model the whole lens profile including the equatorial regions, they did not represent the whole lens profile with a single mathematical expression. This limitation caused, in the TCM model, the lens surfaces and their derivates to be discontinuous at the equator and in the OCM model, the derivative of the profile to be non-zero at the poles. Constraints can be set to fitting functions to overcome this problem. However, these conditions may cause the fitting functions to not closely model the lens.

The purpose of the current study is to develop an age-dependent mathematical model of the whole lens profile, using one mathematical function. This model should address the shortcomings of the previous models and ideally satisfy as many as possible of the geometrical conditions defined above. This age-dependent model should provide reliable lens contour shapes to facilitate development of new mathematical functions for various purposes such as optical modeling and analytical and numerical modeling of the lens.

2. Materials and methods

2.1. Lens preparation

All human eyes were obtained and used in compliance with the guidelines of the Declaration of Helsinki for research involving the use of human tissue. Crystalline lenses ($n = 30$) from donors in the age range of 20–69 were used in this study. They were extracted from whole, intact cadaver eyes, obtained from various US eye banks. The post-mortem time ranged from 1 to 5 days. The lens extraction procedure consisted of first removing the globe's posterior pole, the cornea and the iris. Then the adherent vitreous was carefully removed, the zonules were cut and the lens was extracted and placed in the imaging cell containing DMEM (Augusteyn, Rosen, Borja, Ziebarth, & Parel, 2006). Of the 105 lenses available for this study, 75 lenses were excluded either because of capsular tear or separation. This proportion is similar to that reported by Augusteyn et al. (2006).

2.2. Lens imaging and image analysis

Lenses were imaged using the technique of shadow-photogrammetry (Augusteyn et al., 2006; Denham, Holland, Mandelbaum, Pflugfelder, & Parel, 1989; Pflugfelder et al., 1992; Rosen et al., 2006; Urs et al., 2009). The shadow-photogrammetric system consists of a modified optical comparator (BP-30S, Topcon, Tokyo, Japan) with two light sources to enable photography of the crystalline lens in the coronal and sagittal planes. A 20× magnified shadow of the excised lens is projected onto a viewing screen and images are captured by a 4.0 Mp Nikon Coolpix 4500 digital camera (Tokyo, Japan) positioned at a fixed distance from the screen. For scaling purposes a ruler (1376T-25, Keuffel and Esser Co., Hoboken, New Jersey) was concurrently photographed on each image.

Lens contour detection has been described in a previous publication (Urs et al., 2009). Binary images of the lens contours were loaded into MATLAB. An approximate center for the lens was determined by examining the outermost pixels along the equatorial axis and the optical axis. The initially centered lens contour was positioned such that, the anterior surface of the lens was in the second and third quadrants of the Cartesian co-ordinate system and the posterior surface in the first and fourth quadrants (Fig. 1). This co-ordinate system was converted to polar domain and the lens contour was fit to a 10th-order cosine series function (Eq. (1)) using MATLAB's curve-fitting toolbox.

$$\rho_c(\theta) = a_0 + \sum_{n=1}^{10} a_n \cos\left(n \times \left(\tan^{-1}\left(\frac{y - y_c}{x - x_c}\right) - \theta_c\right)\right) \quad (1)$$

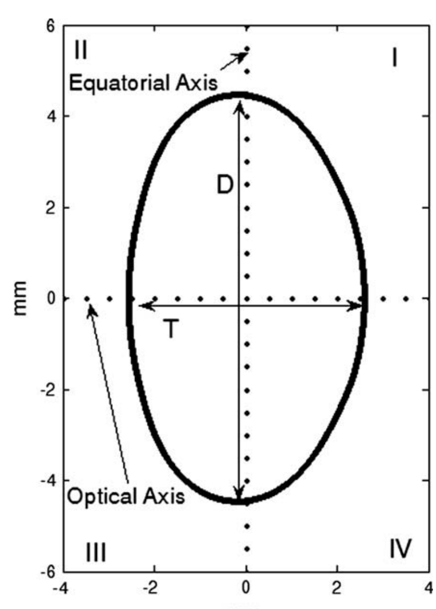

Fig. 1. The co-ordinate system for the Fourier model. The lens anterior surface was placed in quadrants II and III and the posterior surface was placed in the quadrants I and IV. *T* and *D* represent the thickness and diameter of the lens.