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The Riesz transform and simultaneous representations of phase, energy and orientation in spatial vision

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ABSTRACT

To represent the local orientation and energy of a 1-D image signal, many models of early visual processing employ bandpass quadrature filters, formed by combining the original signal with its Hilbert transform. However, representations capable of estimating an image signal's 2-D phase have been largely ignored. Here, we consider 2-D phase representations using a method based upon the Riesz transform. For spatial images there exist two Riesz transformed signals and one original signal from which orientation, phase and energy may be represented as a vector in 3-D signal space. We show that these image properties may be represented by a Singular Value Decomposition (SVD) of the higher-order derivatives of the original and the Riesz transformed signals. We further show that the expected responses of even and odd symmetric filters from the Riesz transform may be represented by a single signal autocorrelation function, which is beneficial in simplifying Bayesian computations for spatial orientation. Importantly, the Riesz transform allows one to weight linearly across orientation using both symmetric and asymmetric filters to account for some perceptual phase distortions observed in image signals – notably one's perception of edge structure within plaid patterns whose component gratings are either equal or unequal in contrast. Finally, exploiting the benefits that arise from the Riesz definition of local energy as a scalar quantity, we demonstrate the utility of Riesz signal representations in estimating the spatial orientation of second-order image signals. We conclude that the Riesz transform may be employed as a general tool for 2-D visual pattern recognition by its virtue of representing phase, orientation and energy as orthogonal signal quantities.

1. Introduction

In many signal processing applications, the 1-D analytic representation of a real-valued function-defined by the linear combination of the original function and its Hilbert transform (Gabor, 1946; Franks, 1968; Papoulis, 1991) — is regarded as an important step because it leads to a complex signal representation from which the phase, energy and (instantaneous) frequency of a 1-D signal may be estimated. Analytic signal representations are also thought to be embedded within neural systems: Marcelja (1980), noting the similarity between receptive field profiles of V1 neurons and symmetric/asymmetric Gabor functions, was an early proponent of this idea. Marcjela's observations inspired a number of computational models of human vision, each designed to provide an economical means of processing the phase and amplitude of 1-D signals using a basis set of symmetric and asymmetric filter profiles (e.g. Daugman, 1985; Morrone, Ross, Burr, & Owens, 1986; Morrone & Burr, 1988).

How one generalizes the 1-D definition of a signal's absolute phase and energy into 2-D has, however, proven to be a challenging problem (Knutsson, 1982; Morrone & Owens, 1987; Morrone & Burr, 1988; Nordberg, 1994; Robbins & Owens, 1997). Some have extended the 1-D definition of the analytic signal into 2-D using orientation and spatial frequency tuned filters arranged in a polar form, such that Hilbert transforms are taken about an axis orthogonal to the preferred orientation tuning of each filter (Daugman, 1985; Freeman & Adelson, 1991; Knutsson, 1982). Computational models arising from a polar decomposition have, however, concentrated on phase independent signal representations. A popular example is the energy model (e.g. Adelson & Bergen, 1985; Knutsson, 1982; Langley & Atherton, 1991; Morrone & Burr, 1988), in which the response of an orientation tuned filter and its Hilbert transform are first squared, then assigned an orientation label from which a spatial orientation vector is estimated. By definition, the energy model gives no information about how a 2-D image signal's spatial phase is represented. However, when detecting an image signal's features, the congruency of spatial phase, especially when

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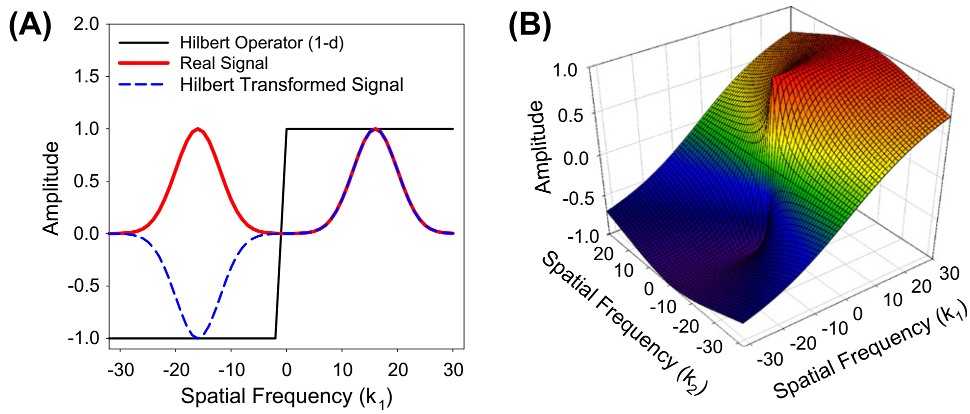

Fig. 1. (A): Shows the Fourier transformed Hilbert operator $\frac{1}{\pi x} \rightarrow^{FT} j\,\frac{k_1}{|k_1|} = j\ \mathrm{sgn}(k_1)$ (black curve), a real signal (red curve) and its Hilbert transform (blue dashed curve). Note that the Hilbert transform may be understood as a flipping in sign of filter sensitivity about the origin in frequency space. The change in sign alters the phase of a signal by $\frac{\pi}{2}$ but the envelope (energy spectrum) for non-zero frequencies is preserved. (B): Illustrates one of the two Riesz transform operators that may be applied to 2-D image signals. In the frequency domain, the Riesz transform is equivalent to a multiplication of an original signal by the operator $j\frac{k_i}{(k_1^2+k_2^2)^{\frac{1}{2}}}$ with $i = 1,2$.

compared across different spatial scales, allows one to distinguish between edges and lines (e.g. Burr, Morrone, & Spinelli, 1989; Canny, 1986; Georgeson, 1992; Georgeson & Meese, 1997; Kovesi, 2000; Watt & Morgan, 1985). While often overlooked, there is little argument that the explicit representation of a signal's 2-D phase may be of utility to vision systems.

Recent work in image processing has advanced a number of promising algorithms capable of extending the 1-D analytic signal into 2-D, thus enabling the computation of an image signal's 2-D phase (Felsberg & Sommer, 2001; Felsberg, 2002; Kovesi, 2000; Mellor & Brady, 2005; Zang & Sommer, 2007). One exciting idea is to consider the Riesz transform as a generalization of the Hilbert Transform. The simultaneous representation of 2-D phase and orientation is made possible because the analytic signal in the Riesz domain is defined by the number of signal dimensions plus one. The purpose of this paper is to introduce the Riesz transform and demonstrate its various benefits in the estimation of: (i) 2-D phase; (ii) phase-dependent and phase-independent spatial orientation vectors; and (iii) orientation defined by second-order signals. We also outline the benefits of Riesz signal representations for Bayesian computations. In presenting our paper, we first define the Riesz transform. We then consider implementations of the Riesz transform insofar as neural systems are concerned, before extolling its computational virtues in the results section.

2. The Hilbert transform

Here, the Hilbert Transform is briefly reviewed before considering its generalization, known as the Riesz transform (Felsberg & Sommer, 2001; Felsberg, 2002; Zang & Sommer, 2007). The Hilbert transform of a 1-D signal $f(t)$, is denoted by $\hat{f}(t)$ and defined by a convolution integral whose interpretation is best understood by taking Fourier transforms ($\rightarrow^{FT}$), as the following steps show:

$$\hat{f}(t) = f(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(\tau)}{t-\tau}\mathrm{d}\tau \rightarrow^{FT} j\ \mathrm{sgn}[\omega]F(\omega) \qquad (1)$$

where the multiplicative function $\mathrm{sgn}[\omega]$, which defines the 1-D Hilbert transform, is depicted in Fig. 1A by the black line. From the far right expression of Eq. (1) note that the Hilbert transform shifts the phase of the original signal by $\frac{\pi}{2}$ radians (Bracewell, 1999). Examples of a real signal and its Hilbert transform are shown in Fig. 1A by the functions $F(\omega)$(Red curve)[1] and $\widehat{F}(\omega)$ (Blue curve), respectively. From the figure, and when excluding the mean of the real signal, note that the frequency support of both the original and Hilbert transformed signals are equal.

2.1. The monogenic signal

The monogenic signal is a representation derived from a generalization of the 1-D Hilbert transform to a higher dimensional signal space (Felsberg & Sommer, 2001; Zang & Sommer, 2007). A generalization is made possible by the Riesz transform (Riesz, 1928), which is defined as:

$$\mathscr{R}[I(x,y)] \rightarrow^{FT} j\ \frac{\mathbf{k}}{|\mathbf{k}|}\widehat{I}(\mathbf{k}) \qquad (2)$$

with $\mathscr{R}[.]$ the Riesz operator, $\mathbf{k} = [k_1, k_2]'$ the signal dimensions in the frequency domain, and $I(x, y)$ the original (untransformed) image signal. Note that the Riesz operator augments spatial image signals by adding two orthogonal Riesz transformed signals. In a similar vein to the Hilbert transform, the Riesz transform may be understood in the frequency domain by the multiplication of the original signal with $j\frac{\mathbf{k}}{|\mathbf{k}|}$. One such Riesz operator is shown in Fig. 1B. Evaluating the Riesz transform about the k_1 axis, where $k_2 = 0$, gives $\mathrm{sgn}(k_1) = \frac{k_1}{|k_1^2+0|^{\frac{1}{2}}}$ which is equal to the Hilbert transform for 1-D signals. This computation can be visualized by tracing along the $k_2 = 0$ contour in Fig. 1B. For a signal space $\mathscr{R}^n$, there exist n Riesz filters. Thus the Riesz transform of a two-dimensional image signal gives a 3-D vector:

$$R(x,y) = \left[\,I(x,y),\ I_{|x|}(x,y),\ I_{|y|}(x,y)\,\right] \qquad (3)$$

where $I(x, y)$ is the original signal and $I_{|x|}(x, y)$, $I_{|y|}(x, y)$ represent the Riesz transformed signals taken about the $|x|$ and $|y|$ axes, respectively.

From the Riesz triple vector $R(x, y)$, an image signal's energy is defined by:

$$E(x,y) = I(x,y)^2 + I_{|x|}(x,y)^2 + I_{|y|}(x,y)^2 \qquad (4)$$

and by definition:

$$\mathscr{E}[I(x,y)^2] = \mathscr{E}[I_{|x|}(x,y)^2] + \mathscr{E}[I_{|y|}(x,y)^2] \qquad (5)$$

where $\mathscr{E}[.]$ denotes the expectation operator. From Eq. (4) note that the Riesz energy is defined by a sum of squares of the three elements of the Riesz triple vector (Felsberg & Sommer, 2001; Felsberg, 2002; Zang & Sommer, 2007). Eq. (5) follows from the definition of

[1] For interpretation of color in Figs. 1, 2, 4, 6, 9, 10 the reader is referred to the web version of this article.