Note on “Hesitant fuzzy prioritized operators and their application to multiple attribute decision making”

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A R T I C L E   I N F O
Article history:
Received 27 November 2014
Revised 3 October 2015
Accepted 27 December 2015
Available online 11 January 2016

Keywords:
Decision making
Hesitant fuzzy sets
Prioritized aggregation operator
Idempotency

A B S T R A C T
Motivated by the idea of prioritized aggregation (PA) operators, Wei (2012) developed two hesitant fuzzy prioritized aggregation (HFPA) operators, and discussed their desirable properties, but the definitions for the HFPA operators and their properties still need to be improved. In this short note, a numerical example is given to show that the idempotency of the HFPA operators suffers from certain shortcomings. Then, based on some adjusted operations on the hesitant fuzzy elements (HFEs), two improved aggregation operators are investigated to aggregate the collective of attribute values. We further prove that the improved operators have the properties of idempotency and boundedness. Finally, the comparison with the method proposed by Wei (2012) is performed to demonstrate that the proposed information aggregation method is both valid and practical to deal with decision making problems.

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1. Introduction

Since Zadeh introduced the fuzzy sets (FSs) \cite{2}, fuzzy sets have been achieved a great success in various fields. The concept of intuitionistic fuzzy sets (IFs) \cite{3} put forward by Atanassov is a generalization of the fuzzy set. Atanassov and Gargov further introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) \cite{4}, the components of which are intervals rather than exact numbers. Torra \cite{5} proposed the concept of hesitant fuzzy sets (HFS) considered as another generalization of FSs, which permits the membership degree having a collection of possible values. Wei \cite{1} extended the prioritized averaging (PA) operator to accommodate the situations where the input arguments are hesitant fuzzy information, and developed two prioritized aggregation operators. He further studied some desirable properties of the proposed operators. However, a close examination demonstrates that some properties suffer from serious drawbacks. The purpose of this note is to point out and correct errors in the properties of HFPA operators.

2. Preliminaries

By the relationship between the HFSs and intuitionistic fuzzy values (IFVs), Xia and Xu \cite{6} defined some operations on the HFSs.

Combined with the PA operator, Wei \cite{1} developed two HFPA operators, including the hesitant fuzzy prioritized weighted average (HFPAW) operator and the hesitant fuzzy prioritized weighted geometric (HFPWG) operator.

\textbf{Definition 1.} (See Wei \cite{1}, Definitions 7 and 8) Let $h_j, j = 1, 2, \ldots, n$ be a collection of HFEs, then the HFPAW operator and the HFPWG operator are defined as follows, respectively:

\begin{equation}
\text{HFPAW}(h_1, h_2, \ldots, h_n) = h_1 \oplus \bigg( \sum_{j=1}^{n} \left( 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{\frac{r_j}{\sum_{j=1}^{n} r_j}} \right) \bigg),
\end{equation}

\begin{equation}
\text{HFPWG}(h_1, h_2, \ldots, h_n) = h_1 \otimes \bigg( \prod_{j=1}^{n} (\gamma_j)^{\frac{r_j}{\sum_{j=1}^{n} r_j}} \bigg),
\end{equation}

where $T_j = 1, T_j = \prod_{j=1}^{n-1} s(h_k), j = 2, \ldots, n$, and $s(h_k)$ is the score values of $h_k, k = 1, 2, \ldots, n$.

Then, Wei \cite{1} proved that both the HFPAW operator and the HFPWG operator are idempotent.

\textbf{Theorem 1.} (Idempotency, see Wei \cite{1}, Theorems 2 and 6) Let $h_j, j = 1, 2, \ldots, n$, be a collection of HFEs, where $T_1 = 1,
Thus, we calculate the score value of $\text{HFPWG}(h_1, h_2)$ and get that

$$s(\text{HFPWG}(h_1, h_2)) = 0.5274,$$

and then

$$s(\text{HFPWG}(h_1, h_2)) = 0.5274 < 0.54 = s(h),$$

which indicates that

$$\text{HFPWG}(h_1, h_2) < h.$$  

Example 1 demonstrates that Theorem 1 of the HFPA operators cannot be tenable, which suffer from serious drawbacks. In this case, the operations on the HFEs need to be improved. In the following section, some adjusted operations for HFEs are presented, and two new HFPA operators are developed, which satisfy the properties of idempotency and boundedness.

4. The improved operators and their properties

In this section, some adjusted operations on the HFEs are reviewed, and two new improved aggregation operators based on these adjusted operations are investigated to aggregate the collective attribute values. We further prove that the improved operators have the properties of idempotency and boundedness. In the end, an example shows that our method is easier than that of Wei [1] in some cases.

Remark 1. Notice that the number of values in different HFEs may be different. Suppose that $h_l$ stands for the number of values in $h$, then the following assumptions are made:

(R1) All the elements in each HFE $h$ are arranged in decreasing order, and $y^{(l)}$ is the $l$th largest value in $h$;
(R2) If $h_l \neq l$, then $l = \max(h_l, l_h)$. To have a correct comparison, the two HFEs $h$ and $g$ should have the same length. If there are fewer elements in $h$ than in $g$, an extension of $h$ should be considered optimistically by repeating its maximum element until it has the same length with $g$;
(R3) For convenience, we assume that all the HFEs have the same length $l$, i.e., $h = \{y^{(1)}, y^{(2)}, \ldots, y^{(l)}\}$.

Now, let’s review some adjusted operations on the HFEs as follows:

Definition 2. [7] Suppose that $h_1$, $h_2$ and $h$ are three HFEs, then

\begin{itemize}
  \item[(1)] $h_1 \oplus h_2 = \bigcup_{j=1}^{l} \{y_1^{(j)} + y_2^{(j)} - y_1^{(j)} y_2^{(j)}\}$;
  \item[(2)] $h_1 \oplus h_2 = \bigcup_{j=1}^{l} \{y_1^{(j)} y_2^{(j)}\}$;
  \item[(3)] $\lambda h = \bigcup_{j=1}^{l} \{1 - (1 - y^{(j)})^\lambda\}, \lambda > 0$;
  \item[(4)] $h^2 = \bigcup_{j=1}^{l} \{(y^{(j)})^2\}, \lambda > 0$.
\end{itemize}

According to the Definition 2, we know that $h_1$, $h_2$, $h_1 \oplus h_2$ and $h_1 \oplus h_2$ have the same length $l$.

Example 2. Suppose that $l = 3$, $\lambda = 2$, $h_1$ and $h_2$ are two HFEs, and $h_1 = [0.77, 0.50, 0.35], h_2 = [0.86, 0.59, 0.22]$, then we have

\begin{itemize}
  \item[(1)] $h_1 \oplus h_2 = \bigcup_{j=1}^{l} \{y_1^{(j)} + y_2^{(j)} - y_1^{(j)} y_2^{(j)}\} = [0.9678, 0.7950, 0.4930]$;
  \item[(2)] $h_1 \oplus h_2 = \bigcup_{j=1}^{l} \{y_1^{(j)} y_2^{(j)}\} = [0.6622, 0.2950, 0.0770]$;
  \item[(3)] $\lambda h = \bigcup_{j=1}^{l} \{1 - (1 - y^{(j)})^\lambda\}, \lambda > 0$;
  \item[(4)] $h^2 = \bigcup_{j=1}^{l} \{(y^{(j)})^2\} = [0.5929, 0.2500, 0.1225]$.
\end{itemize}

Based on the adjusted operational principle for HFEs, we investigate the improved HFPA operators as follows: