Vision Research 50 (2010) 330–336

Contents lists available at ScienceDirect

Vision Research

journal homepage: www.elsevier.com/locate/visres

A geometric model of ocular accommodation

Matthew A. Reilly *, Nathan Ravi

Department of Veterans Affairs Medical Center, St. Louis, MO, USA
Department of Energy, Environmental, and Chemical Engineering, Washington University in St. Louis, St. Louis, MO, USA
Department of Ophthalmology and Visual Sciences, Washington University in St. Louis, St. Louis, MO, USA

ARTICLE INFO

Article history:
Received 5 June 2009
Received in revised form 22 November 2009

Keywords:
Accommodation
Presbyopia
Lens
Optomechanics
Lens stretching
Lens curvature

ABSTRACT

An exoteric geometric mechanics model of ocular accommodation is detailed to elucidate the main ideas of various ongoing modeling efforts. The present study derives solutions for the stretched state of the ocular lens as it might appear during accommodation by using simple geometric arguments and a volume constraint, rather than the more mathematically intensive theory of elasticity. Results show that geometric shapes similar to the lens will deform in a similar fashion. This implies that, while the true lens geometry is somewhat more complex, it should also follow these qualitative behaviors.

Published by Elsevier Ltd.

1. Introduction

Accommodation is the ability of the eye to change its focal distance from far to near. von Helmholtz (1855) stated that the optical power of the lens decreases as a result of increased equatorial diameter. Presbyopia – the loss of accommodation amplitude with age – has been investigated as extensively as accommodation itself. Its pathogenesis is likely multifactorial, resulting from changes in lens mechanical properties (Fisher, 1971; Weeber & van der Heijde, 2007), lens volume (Sakabe, Oshika, Lim, & Apple, 1998), lens geometry (Fisher, 1969; Strenk, Strenk, Semmlow, & DeMarco, 2004), extralenticular anatomy, or a combination of these factors (Weale, 1989).

A litany of experimental support has been obtained for the Helmholtz theory of accommodation (e.g. Glasser & Kaufman, 1999; Hermans et al., 2009; Koretz, Handelman, & Brown, 1984; Pierscionek, 1993; Reilly, Hamilton, & Ravi, 2008; Strenk et al., 1999, 2004). Extensive mechanical modeling of the accommodative system has also been undertaken (Chien, Huang, & Schachar, 2006; Koretz & Handelman, 1982, 1986), with most recent works utilizing finite element analysis (Burd, Judge, & Flavell, 1999, 2002; Belaidi & Pierscionek, 2007; Weeber & van der Heijde, 2007). However, these models are not generally accessible to the broader ophthalmic community due to the use of the specialized language of mechanics. These models also require detailed mechanical properties as input data. The available data for human lenses (Fisher, 1971; Heys, Cram, & Truscott, 2004; Weeber et al., 2005; Weeber & van der Heijde, 2007) have been questioned due to both modeling assumptions and treatment of the lens tissue prior to testing (Burd, Wilde, & Judge, 2006; Schachar, 2005, 2007), implicitly calling the results of these models into question. We have recently published data on the mechanical properties and optomechanical performance of fresh 6-month-old porcine lenses (Reilly & Ravi, 2009; Reilly, Hamilton, Perry, & Ravi, 2009), though no mechanical model has yet been developed utilizing these data.

Therefore, we propose a model based solely on geometric parameters that may be readily understood and used to analyze the role of changes in lens geometry with age as a potential cause of presbyopia. This model is exoteric and should be accessible for the larger ophthalmic community. Further, its only required input data are the equatorial radius and axial thickness of the lens, which are well known from experimental observations. This model computes changes in optical parameters which occur due to changes in lens equatorial diameter assuming constant lens volume.

2. Methods

2.1. Geometric descriptions

The mechanical models of lens stretching available in the literature utilize a variety of geometric descriptions for the lens. We assumed that the lens is symmetric about the optical axis. Thus, the lens may be completely described as a surface of revolution

* Corresponding author. Address: Washington University in St. Louis, 660 S. Euclid Ave., Box 8096, St. Louis, MO 63110, USA.
E-mail address: reilly@vision.wustl.edu (M.A. Reilly).

0042-6989/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.visres.2009.11.020

by specifying only the radial coordinate r and axial coordinate z, which gives the cross-sectional profile of either the anterior or posterior surface of the lens. We assume that the lens must maintain the same geometric shape class (i.e. an ellipsoid lens must remain an ellipsoid after stretching), and that the volume remains constant during stretching (Hermans et al., 2009). Each geometric shape class has a corresponding radius of curvature, surface area, and volume which are geometrically related to its profile $z(r)$.

The initial equatorial radius a and axial thickness t for the human lens were taken as the average measured for 29-year-old lenses as measured by Strenk et al. (1999) and Dubbelman et al. (2005): 4.40 mm and 2.01 mm (half of the thickness of the whole lens), respectively, for the human lens. Note that Strenk et al. measured the fully accommodated thickness as 3.96 mm, which is sufficiently close to Dubbelman et al.'s result as to make no discernable difference in the results. The initial parameters of the six-month-old porcine lens were taken from Reilly et al. (2009): 5.02 mm and 3.93 mm for the equatorial radius and axial half-thickness, respectively.

2.1.1. Spherical cap

The simplest geometry which may describe a lens-like object is the spherical cap, which is simply a truncated sphere (Fig. 1A). The functional form of its profile is given by

$$z(r) = t - R + \sqrt{R^2 - r^2}, \tag{1}$$

where a is the equatorial radius of the lens and t corresponds to the half-thickness of the lens. The radius of curvature R of the spherical cap is uniform everywhere and is given by

$$R = \frac{a^2 + t^2}{2t}. \tag{2}$$

The surface area S is given by

$$S = 2\pi Rt. \tag{3}$$

The volume V of the spherical cap is given by

$$V = \frac{\pi}{6} t(3a^2 + t^2). \tag{4}$$

2.1.2. Paraboloid

The paraboloid (Fig. 1B) is similar to a spherical cap, though perhaps with a slightly more realistic profile since R increases with the radial coordinate r. The generating function is

$$z(r) = t\left(1 - \frac{r^2}{a^2}\right), \tag{5}$$

and the radius of curvature is given by

$$R(r) = \frac{(1 + 4t^2r^2/a^4)^{3/2}}{2t/a^2}. \tag{6}$$

The reported values for R were computed by averaging the result of Eq. (6) at 100 evenly spaced points within the optical zone (i.e. within 1.5 mm radius of the optical axis). The surface area S is given by

$$S = \frac{\pi a}{6t^2}\left[(a^2 + 4t^2)^{3/2} - a^3\right]. \tag{7}$$

The volume V is given by

$$V = \frac{\pi}{2} a^2 t. \tag{8}$$

2.1.3. Oblate spheroid

The spherical cap and paraboloid both exhibit discontinuity in curvature at the equator. Therefore, this geometry is generally not suitable for mechanical modeling purposes. The slightly more complex oblate hemispheroid (Fig. 1C) gives a fairly accurate fit for the lens surfaces, where each surface has a minor radius equal to its thickness, t. The major radius, which corresponds to the

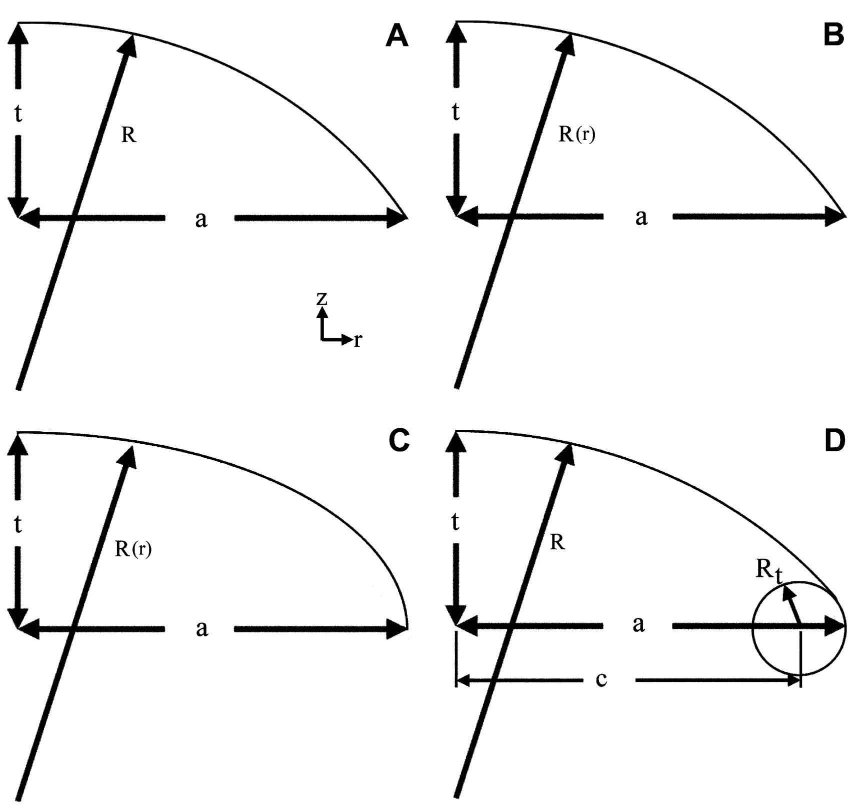

Fig. 1. Depiction of the cross-sections of (A) spherical cap, (B) paraboloid, and (C) oblate spheroid geometries, and (D) torispherical dome.