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An approach to determining the integrated weights of decision makers based on interval number group decision matrices

Wei Liu, Lei Li*

Business School, Jiangnan University, Wuxi, Jiangsu 214122, China

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ABSTRACT

In this paper, we develop an approach to determining the integrated weights of decision makers (DMs) with interval numbers in multiple attribute group decision making (MAGDM) problems. We first map the interval numbers of each DM's decision matrix into two-dimensional coordinates. The interval number values correspond to the coordinate values one to one. By integrating up-front subjective weight assignment with the relative importance of the DMs simultaneously, we derive the adjusted subjective DM weights. Based on the adjusted subjective weights, a plant growth simulation algorithm (PGSA) is used to find the generalized Fermat–Torricelli point of every point set, i.e., the optimal rally points that reflect the preferences of the DM group as a whole. From the mapping relationship, the generalized Fermat–Torricelli points constitute the ideal interval number decision matrix. Using deviation distance between each DM's decision matrix and the DM group's ideal matrix, we then obtain the degree of similarity indexes of the DMs. Next, by normalizing the degree of similarity indexes, we calculate the objective DM weights. Finally, we derive the stable integrated DM weights by combining the adjusted subjective weights and the objective weights. In addition, a numerical example is provided to illustrate the efficiency and reasonableness of the proposed approach.

1. Introduction

Multiple attribute group decision making (MAGDM) problems have been the object of numerous studies, with applications in such diverse fields as economics, management, and military research [1–6]. In MAGDM analysis, each decision maker (DM) considers his or her own preferences and provides judgment information on possible alternatives over a range of attributes. Each DM's judgment information is then aggregated by a given method to form an overall ranking of possible alternatives.

When dealing with judgment information in MAGDM problems, no matter what method is adopted, the key issue is how to determine the respective weight given to each DM. A DM's weight reflects his or her importance. The greater a DM's overall competence, the higher the weight should be. DM weights can be divided into subjective weights and objective weights. Subjective weights are derived from known information, such as a DM's fame, status, profession, and level of familiarity with the issue at hand, and so on. In addition to well-known approaches to obtaining subjective weights, such as AHP and Delphi [7], Bodily [8] derives DM weights as a result of a delegation process in which each DM designates voting weights in a subcommittee made up of other group DMs. Ramanathan and Ganesh [9] put forward the method of mutual evaluation among DMs, which requires the DMs in the group to have a high level of familiarity with each other. Van den Honert [10] adapts the REMBRANDT suite of decision models (multiplicative AHP and SMART) to measure decisional power in groups, and generalizes this to explain cases where power itself is deemed to be multidimensional in nature, as well as cases where subjective judgment of power among group members are uncertain.

Objective weights are obtained by quantitatively judging the quality of each DM's judgment information. The literature provides many approaches to deriving objective DM weights. For example, Xu [11] presents two methods for deriving DM weights based on decision matrices and error analysis. Chen and Fan [12] propose a factor score method (FAM) to obtain a ranking of the assessment levels of DMs in group decision analysis. Chen et al. [13] provide a new analytical solution to assess the level of DMs on the basis of mathematical statistics theory. Xu and Cai [14] aggregate all of the individual decision matrices into a collective decision matrix by means of a weighted arithmetic averaging operator. The authors then establish a general nonlinear optimization model and employ a genetic algorithm (GA) to find the optimal DM weights. Yue establishes an approach to determining DM weights in MAGDM problems using an extended TOPSIS in group settings with crisp numbers [15] and interval numbers [16,17]. In a later paper, Yue [18] develops an

* Corresponding author. Tel.: +86 13921286093.
E-mail address: inf2007@126.com (L. Li).

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approach to determining DM weights in a group decision environment based on the projection method, comparing it with an extended TOPSIS method. Yue [19] further aggregates all individual decisions by averaging them to generate a collective decision deemed to be the positive ideal decision (PID). He then uses the Euclidean distances between the individual decisions and the ideal decision to derive the DM weights. More recently, Zhang [20] presents a novel approach to determining DM weights in intuitionistic fuzzy group decision making based on consistency maximization by simultaneously considering the ranking and magnitude of decision information. In summary, the above-mentioned methods of determining objective DM weights rely on each DM's degree of consensus with the DM group: the closer a DM's judgment information is to the group consensus, the greater the corresponding weight of the DM.

Another common method of determining objective DM weights is based on clustering analysis, which usually divides DMs into several clusters based on preferences or evaluation information to reflect the degree of similarity among the DMs [21,22], thereby reducing the complexity of the computation process and making the information aggregation process more accurate. In a brief review of the literatures, Zahir [23] discusses and implements an algorithm to group individuals into natural clusters using a convenient similarity measure. Liu et al. [24] provide an efficient approach to DM clustering based on an interval-valued intuitionistic fuzzy setting. Liu et al. [25] propose a two-layer weight determination model to objectively obtain DM weights in a linguistic environment in which all of the DMs' clustering results are known.

Some studies propose methods that take into account both subjective and objective weight, synthesizing them into an integrated weight [26–30]. In particular, Liu et al. [29] propose an adaptive adjustment weighting method for MAGDM problems based on the given subjective weights of DMs and attributes, which employs entropy coefficient theories and compares the difference between individual and group decision results to adjust weights to make these weights more reliable. Chen and Liu [30] design an adaptive iterative algorithm for MAGDM problems, achieving stable results for objective and integrated DM weights after several iterations.

In general, the literature regarding DM weight determination in MAGDM problems has developed by investigating definitions or methods. Gaps still remain, however, calling for further research. For example, studies aggregating individual DM judgment information into collective DM group information commonly use information aggregating operators such as a weighted arithmetic averaging (WAA) operator, a weighted geometric averaging (WGA) operator, or an ordered weighted geometric averaging (OWGA) operator. However, the collective DM group information derived by these methods may become overall average and ignore some information that deviates from the average level. The aggregation method of DM judgment information has a critical impact on the subsequent weight assessment of the DMs. We must therefore improve existing aggregation methods.

In some real-life situations, DMs may not be able to express the values they assign to attributes accurately, instead expressing them only as intervals. Therefore, in this paper, each DM provides his or her judgment information on alternatives with respect to attributes in the form of interval numbers [31,32], thus generating an individual decision matrix. We propose an approach to determining integrated DM weight based on interval number group decision matrices.

The remainder of this paper is set out as follows: Section 2 reviews the definition and usage of interval numbers. Section 3 discusses the weighted Fermat–Torricelli problem and the plant growth simulation algorithm (PGSA), including mapping the interval numbers into the corresponding planar point sets and aggregating the individual decision matrices into the ideal decision matrix by PGSA. Section 4 addresses the methods and procedures of deriving integrated DM weights. In Section 5, a numerical example is given to illustrate the efficiency and reasonableness of the proposed approach. Section 6 offers some conclusions and suggestions for future research.

Table 1
Decision matrix with interval numbers of rth DM.

	E_1	E_2	$\cdots$	E_n
S_1	$[a^r_{11}, b^r_{11}]$	$[a^r_{12}, b^r_{12}]$	$\cdots$	$[a^r_{1n}, b^r_{1n}]$
S_2	$[a^r_{21}, b^r_{21}]$	$[a^r_{22}, b^r_{22}]$	$\cdots$	$[a^r_{2n}, b^r_{2n}]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
S_m	$[a^r_{m1}, b^r_{m1}]$	$[a^r_{m2}, b^r_{m2}]$	$\cdots$	$[a^r_{mn}, b^r_{mn}]$

2. Interval numbers

Definition 1 [33]**.** Let z be a nonnegative interval number, which has the following form:

$z = [a, b] = \{x | 0 \le a \le x \le b\}$. If $a = b$, z is degenerated into a nonnegative real number.

NOTE: For computational convenience, throughout this paper all of the interval arguments are nonnegative interval numbers.

In a MAGDM problem, suppose there are p DMs ($D = \{D_1, D_2, \ldots, D_p\}$) who are asked to provide judgment information on m alternatives ($S = \{S_1, S_2, \ldots, S_m\}$) over n attributes ($E = \{E_1, E_2, \ldots, E_n\}$), and their subjective weights are $w_1, w_2, \ldots, w_p$, such that their decision matrices with interval numbers can be concisely expressed in the format as follows (see Table 1):

$$A^{(r)} = \left([a^r_{ij}, b^r_{ij}]\right)_{m\times n} \tag{1}$$

for all $r = 1, 2, \ldots, p$; $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

3. The aggregation of interval number decision matrices

3.1. The mapping of interval numbers in decision matrices

In order to facilitate the aggregation of individual judgment information, we map the interval numbers in the decision matrices into two-dimensional coordinates. If the random interval numbers $[a^r_{ij}, b^r_{ij}]$($r = 1, 2, \ldots, p$; $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$)are mapped into two-dimensional coordinates, a^r_{ij} is the abscissa value and b^r_{ij} is the ordinate value. So, a MAGDM problem with interval numbers, which contains p DMs and m alternatives with respect to n attributes, can be regarded as p planes which contain $m \times n$ planar point sets:

$$A^{(r)} = [a^r_{ij}, b^r_{ij}] \to (a^r_{ij}, b^r_{ij}) \in R^2 \tag{2}$$

for all $r = 1, 2, \ldots, p$; $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$, where R^2 is two-dimensional space.

Expanding Eq. (2) into matrix form yields the following:

$$A^{(r)} = ((a^r_{ij}, b^r_{ij}))_{m\times n} = \begin{pmatrix} (a^r_{11}, b^r_{11}) & (a^r_{12}, b^r_{12}) & \cdots & (a^r_{1n}, b^r_{1n}) \\ (a^r_{21}, b^r_{21}) & (a^r_{22}, b^r_{22}) & \cdots & (a^r_{2n}, b^r_{2n}) \\ \vdots & \vdots & \cdots & \vdots \\ (a^r_{m1}, b^r_{m1}) & (a^r_{m2}, b^r_{m2}) & \cdots & (a^r_{mn}, b^r_{mn}) \end{pmatrix}$$

for all $r = 1, 2, \ldots, p$.

3.2. The aggregation of interval number decision matrices

3.2.1. Weighted Fermat–Torricelli problem

In early 1643, Fermat posed the following mathematical problem: given three random points P_1, P_2, P_3 on a plane, find a fourth point P such that the sum of its Euclidean distances to the three given points