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An analysis of fully fuzzy linear programming with fuzzy decision variables through logistics network design problem

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ABSTRACT

Recently, there is a growing attention by the researchers to solve and interpret the analysis of fully fuzzy linear programming problems in which all of the parameters as well as the decision variables are considered as fuzzy numbers. Under a fully uncertain environment where all of the data are stated as fuzzy, presenting the reasonable range of values for the decision variables may be comparatively better than the currently available crisp solutions so as to provide ranges of flexibility to decision makers. However, there is still a scarcity of solution methodologies on fuzzy mathematical programs with fuzzy decision variables. Based on this motivation, a new parametric method which is mainly based on α-cut representation of fuzzy intervals is proposed in this paper by incorporating the decision maker's attitude toward risk. In order to illustrate validity and practicality of the proposed method, it is applied to a generic reverse logistics network design model including fuzzy decision variables. To the best of our knowledge, this is the first study in the literature which presents fuzzy efficient solutions and analysis for a fully fuzzy reverse logistics network design problem with fuzzy decision variables. The provided solutions by the proposed method are also compared to the available solution methodologies from the literature in terms of computational efficiency, solution quality and ease of use. By using the proposed method, the decision makers can be supported by yielding fuzzy efficient solutions under different uncertainty levels and risk attitudes. The computational results have also shown that more reliable and necessarily precise solutions can be generated by the proposed method for a risk-averse decision maker.

1. Introduction

In the literature, most of the fuzzy mathematical programming models deal with the fuzziness related to the aspiration levels of the objective(s) and some of the model parameters, i.e., the objective function(s)' coefficients, technological coefficients and right hand side values of the constraints. Because, available information on these parameters may not be precise and/or the decision maker(s) generally may not precisely know the values of these parameters. On the other hand, fully fuzzy linear programming (FFLP) problems in which all of the parameters and decision variables are stated as fuzzy numbers have been an attractive topic for the researchers in recent years. However, the literature on fuzzy mathematical programming with fuzzy decision variables is not rich as there are a few studies available on this research topic. In fact, decision variables of a fuzzy mathematical program where all of the parameters are stated as fuzzy should also be considered as fuzzy numbers. Because, the fuzzy characteristic of the decision may be partially lost and the decision making process is limited with the crisp solutions when the decision variables of this problem are crisp [5]. For this reason, instead of crisp solutions, obtaining fuzzy efficient solutions which provide ranges of flexibility to decision maker(s) seems more impressive in fully uncertain environments [15]. In other words, presenting the reasonable range of values for the decision variables can be comparatively better than the currently available crisp solutions [36]. In addition, fuzzy solutions may serve the regions containing potential satisfactory solutions around the optimal solutions to the decision maker(s). Thus, the final decisions can be made by the decision maker(s) as crisp ones. By taking the decision variables as fuzzy, the choice of the crisp decisions among the fuzzy solutions will also be supported [34]. Furthermore, it was emphasized by Xiaozhong et al. [40] that it is possible to encounter some optimization problems such as the cardinality of optimal solutions and the number of fairly superior solutions for a fuzzy linear program. Nevertheless, achieving the optimal solutions of a FFLP problem is a challenging task since there is no algorithm which determines the exact values of the optimal fuzzy decision variables [10].

In this study, a fully fuzzy reverse logistics (RL) network design problem whose parameters as well as the decision variables are taken

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as fuzzy numbers is discussed. Because, the RL network design problems are surrounded by many uncertainties on product returns, capacity of the recovery facilities, disposal and recovery rates. Actually, these are the main essential factors contributed to the uncertainty in RL environments [42]. In other words, uncertainty in timing, quality and quantity of product returns are important aspects of RL network design problems. The decision variables of a fully fuzzy RL network design problem should also be stated as fuzzy numbers because of the following reasons:

- There are two important sources of uncertainty regarding the collection process in a RL network design problem. The first one is the uncertainty related to the quantity of returned products from customer zones and the second one is the capacity of collection/inspection centers. Moreover, uncertain amounts of the collected products may be lost or perished because of the several reasons during the transportation process. Thus, collection quantities should be stated as fuzzy decision variables in a RL network design model.
- Similarly, quantities of recyclable and disposal products should also be defined as fuzzy decision variables due to the uncertainties regarding disposal rate, conformity/acceptance ratio, recycling and disposal capacities.
- Finally, the amounts of recycled materials/components should also be stated as fuzzy decision variables since including some sort of ambiguity on the recovery rates.
- Since the output of any stage in a RL network constitutes the inputs of the consecutive stages, fuzziness in one of the decision variables will also cause uncertainty into the other decision variables. For instance, if the collection quantities are defined as fuzzy, the amounts of recyclable and disposed products should also be stated as fuzzy. Because, the output of the collection stage will be used as inputs in the recovery and disposal stages. Therefore, the decision variables of these stages inherently contain uncertainty in nature.

Furthermore, while realizing the design of a real-life RL networks, it may be possible to perform a lot of revisions for the product flow quantities between the different stages due to the fully uncertain environment and dynamism. For these reasons, producing fuzzy efficient solutions at first will be more appropriate for the decision maker(s) by taking the decision variables, i.e., distribution, collection, recycling, disposal amounts as fuzzy. Besides, the objective function value of a FFLP problem will also be fuzzy in nature since it comprises various fuzzy decision variables. Thus, the decision maker(s) can easily recognize the changes in the objective value when these variables took different crisp values. In other words, the decision maker(s) can maintain an opinion about the range of objective value.

Most of the existing methods for solving FFLP problems are based on the standard fuzzy arithmetic operations and/or Zadeh's extension principle which is one of the most important tools in fuzzy set theory. However, standard fuzzy arithmetic may produce risky solutions and questionable results for real world cases [19]. Thus, generated solutions by the former methods may not be appropriate for a risk-averse decision maker(s). Almost none of the existing methods for solving FFLP problems produce fuzzy solutions according to the risk tolerance of the decision maker(s). Based on this research motivation, the main contribution of this paper is to develop a new parametric method which is able to generate fuzzy efficient solutions for FFLP problems under different uncertainty levels and risk attitudes. Therefore, risk-free and more reliable solutions can be provided according to the attitude of decision maker toward risk. Moreover, since the proposed method is based on α-cuts (it is an alpha parametric model), it is possible to consider/model risk tolerances under various uncertainty levels. Additionally, it is also aimed to present and solve a FFLP model of a generic reverse logistics (RL) network design problem with fuzzy decision variables. In fact, the performance of the proposed method is tested on a RL network design model whose parameters and decision variables are taken as fuzzy numbers. In order to show the validity and practicality of the proposed method, comparative analyses are also conducted with the existing methods in the literature. We also presented the generalized closed-form mathematical representations of the crisp equivalent RL network design model which is transformed by using these methods.

The rest of the paper is organized as follows. In Section 2, a comprehensive literature review and classification on FFLP problems is presented. In Section 3, details of the fully fuzzy RL network design problem with notations, model assumptions and mathematical formulation are described. The fundamentals of the proposed method and the comparative methods to FFLP problems are introduced in Section 4. In the same section, the crisp equivalent forms of the present fully fuzzy RL network design model by these methods are also given. In Section 5, an example problem is illustrated for the solution of a fully fuzzy RL network design problem with extensive comparisons. Finally, Section 6 presents the conclusions, discussions and possible future directions.

2. Literature review on fuzzy mathematical programming with fuzzy decision variables

The concept of fuzzy mathematical programming with fuzzy decision variables was first introduced by Tanaka and Asai [37]. They provided the fuzzy solution of a FLP problem with fuzzy satisfaction criteria and fuzzy right hand side values via maximizing the possibility distributions of the fuzzy decision variables. Xiaozhong [41] obtained the fuzzy optimal solution of a multi-objective FLP problem with fuzzy constraints and variables based on multiple objective optimization and two-stage method. Maleki et al. [27] proposed a method based on the concept of comparing fuzzy numbers. Buckley and Feuring [10] proposed a direct search technique by employing an evolutionary algorithm to solve FFLP problems. In their approach, they first transformed the FFLP model into a multi-objective model. Then, an evolutionary algorithm was designed to find non-dominated solutions. Baykasoglu and Gocken [5] developed a direct solution approach for solving mathematical programming problems with fuzzy decision variables based on integrating a fuzzy ranking method into a meta-heuristic algorithm which aims to rank the objective function values and determine the feasibility degrees of the constraints. Mahdavi-Amiri and Nasseri [26] introduced a new dual algorithm for direct solution of the FLP problems with the help of the duality results and primal simplex tableau. On the other hand, their algorithm is able to solve FLP problems in which the decision variables, right hand side values of the constraints and aspiration levels of the objectives are defined as trapezoidal fuzzy numbers.

Stanciulescu et al. [34] proposed a method which uses fuzzy decision variables with a joint membership function. In that method, the lower boundaries of the decision variables were considered at first. Then, the proposed method was extended to achieve the lower and upper bounded fuzzy variables. Defuzzification of the objectives and constraints was carried out by making use of area compensation method and a worst case approach, respectively. Solution of the crisp equivalent model was provided by an interactive and iterative MAUT method. A two-phase method was developed by Hashemi et al. [15] for solving FFLP problems where all of the parameters and decision variables are described as symmetric fuzzy numbers. Maximizing the possibilistic mean value of the objective function was achieved in the first phase. Then, the standard deviation of the original fuzzy objective function was minimized by using the basic feasible solutions obtained from the first phase. A ranking procedure which depends on total ordering of fuzzy numbers which utilizes non-algebraic numbers was proposed in [14] for solving fully fuzzy minimal cost flow problem.