Knowledge-Based Systems 80 (2015) 122–130

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Intuitionistic fuzzy logics as tools for evaluation of Data Mining processes

Krassimir Atanassov [*,1]

Department of Bioinformatics and Mathematical Modelling, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 105, Sofia 1113, Bulgaria
Intelligent Systems Laboratory, Prof. Asen Zlatarov University, Bourgas 8000, Bulgaria

ARTICLE INFO

Article history:
Received 28 October 2014
Received in revised form 10 January 2015
Accepted 26 January 2015
Available online 2 February 2015

Keywords:
Artificial intelligence
Data Mining
Intuitionistic fuzzy estimation
Intuitionistic fuzzy logics
Intuitionistic fuzzy set

ABSTRACT

The Intuitionistic Fuzzy Sets (IFSs), proposed in 1983, are extensions of fuzzy sets. Some years after their introduction, sequentially, intuitionistic fuzzy propositional logic, intuitionistic fuzzy predicate logic, intuitionistic fuzzy modal logic and intuitionistic fuzzy temporal logic have been introduced, presented here shortly. During the last 25 years, different intuitionistic fuzzy tools have been used for evaluation of objects from the area of the Artificial Intelligence, as expert systems (having, e.g. facts and rules, with intuitionistic fuzzy degrees of validity and non-validity), decision making processes (having, e.g. intuitionistic fuzzy estimations of the criteria), neural networks, pattern recognitions, metaheuristic algorithms, etc. Short review of these legs of research is offered, with some concrete ideas of possible new directions of study. On this basis, a non-formal discussion is raised on the benefits of applying various elements of intuitionistic fuzzy logics as tools for evaluation of Data Mining processes.

1. Introduction

This paper discusses the origin, current state of research and applications in the area of Data Mining (DM) of one extension of fuzzy sets and logic, called Intuitionistic Fuzzy Set (IFS) and Logics (IFLs).

The first researches, related to IFLs started in 1983 together with the researches on IFSs, but the first publications in this area were dated 1988–1990. In them, shortly, ideas for intuitionistc fuzzy propositional calculus [5], intuitionistic fuzzy predicate logic [8], intuitionistic fuzzy modal logic [6] and temporal intuitionistic fuzzy logic [9] are introduced. During the following 25 years, there areas were essentially extended. A lot of operations and operators were defined, but up to now there is not an altogether and systematic description of the obtained results. The present paper contains some basic ideas and some unsolved problems in the area of IFLs that will develop this part of fuzzy sets theory.

The IFSs [13,21] are defined as extensions of the ordinary fuzzy sets [115], but over them, a lot of operators are defined, that do not exist in the theories of the other types of sets. These operators have analogous in IFLs.

The additional component in the IFS- and IFL-definitions give more and larger evaluating possibilities and determine the place of the IFSs and IFLs among the separate types of fuzzy sets. In the last twenty-five years the IFSs are being used for evaluating of processes in a lot of areas, e.g. of Systems Theory, Artificial Intelligence (AI) and Intelligent Systems, medicine, chemical industry, ecology, etc.

Here we give the basic notions from the area of IFLs, describe some its applications in the AI and the benefits of this, and discuss the possibilities for application of the IFLs as tools for evaluating of Data Mining-processes.

2. Short remarks on intuitionistic fuzzy propositional calculus

To each proposition (in the classical sense, see, e.g., [73]) we can assign its truth value: truth – denoted by 1, or falsity – 0. In the case of fuzzy logic this truth value is a real number in the interval $[0,1]$ and may be called "truth degree" of a particular proposition. Here we add one more value – "falsity degree" – which will be in the interval $[0,1]$ as well. Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to the proposition p with the following constraint to hold: $\mu(p)+\nu(p) \leqslant 1$.

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p), \nu(p) \rangle.$$

* Address: Department of Bioinformatics and Mathematical Modelling, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., Bl. 105, Sofia 1113, Bulgaria.

E-mail address: krat@bas.bg

[1] IFSA fellow.

http://dx.doi.org/10.1016/j.knosys.2015.01.015
0950-7051/

Hence the function $V : S \to [0,1] \times [0,1]$ gives the truth and falsity degrees of all propositions in S.

We assume that the evaluation function V assigns to the logical truth $T : V(T) = \langle 1, 0 \rangle$, and to the logical falsity F: $V(F) = \langle 0, 1 \rangle$.

Similarly to IFSs theory (see, e.g., [13,21], several geometrical interpretations of the results of the function V will be discussed below. It is obvious, that the ordinary fuzzy sets have only one geometrical interpretation, while in IFSs case, some geometrical interpretations are given.

The first one (which is analogous to the standard fuzzy set interpretation) is shown in Fig. 1. Its analogue is given in Fig. 2. Therefore we can map to every proposition $p \in S$ a unit segment of the form from Fig. 3.

Another geometrical interpretation of the elements of S is given in [7] (see Fig. 4). It uses the triangle with analytical form $\{\langle x, y \rangle \mid x, y \in [0,1] \& x + y \leqslant 1\}$, that there is called an intuitionistic fuzzy interpretational triangle.

When the values $V(p)$ and $V(q)$ of the propositions p and q are known, the evaluation function V can be extended also for the operations "&", "$\vee$" through different (by the moment – three) definitions (see [5,13,82]):

$$V(p \&_1 q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q)) \rangle,$$
$$V(p \vee_1 q) = \langle max(\mu(p), \mu(q)), min(\nu(p), \nu(q)) \rangle;$$
$$V(p \&_2 q) = \langle \mu(p).\mu(q), \nu(p) + \nu(q) - \nu(p).\nu(q) \rangle,$$
$$V(p \vee_2 q) = \langle \mu(p) + \mu(q) - \mu(p).\mu(q), \nu(p).\nu(q) \rangle;$$
$$V(p \&_3 q) = \langle \min(1, \mu(p) + \mu(p)), \max(0, \nu(p) + \nu(p) - 1) \rangle,$$
$$V(p \vee_3 q) = \langle \max(0, \mu(p) + \mu(p) - 1), \min(1, \nu(p) + \nu(p)) \rangle.$$

Everywhere below we shall assume that for the two variables p and q there hold the equalities: $V(p) = \langle a, b \rangle, V(q) = \langle c, d \rangle, (a, b, c, d, a + b, c + d \in [0, 1])$.

For the needs of the discussion below, we shall define the notion of Intuitionistic Fuzzy Tautology (IFT, see [5,21]) by:

p is an IFT if and only if $a \geqslant b$,

while p will be a tautology iff $a = 1$ and $b = 0$.

In some definitions, we use functions sg and $\overline{\text{sg}}$ defined by,

$$\text{sg}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leqslant 0 \end{cases}, \quad \overline{\text{sg}}(x) = \begin{cases} 0 & \text{if} x > 0 \\ 1 & \text{if} x \leqslant 0 \end{cases}$$

In a series of papers of the author, starting with [5,6], 138 different intuitionistic fuzzy implications and 34 different intuitionistic fuzzy negations, generated by the intuitionistic fuzzy implications were defined and some of their basic properties were studied. Meantime, in [30–32], Lilija Atanassova introduced implications $\to_{139}, \ldots, \to_{149}$ and negations $\neg_{35}, \ldots, \neg_{41}$; and in [48–50], Piotr Dworniczak introduced implications $\to_{150}, \ldots, \to_{152}$ and negations $\neg_{42}, \ldots, \neg_{45}$. The author introduced also implication $\to_{153}$ and negation $\neg_{46}$ in [16].

The list of all existing at the moment intuitionistic fuzzy implications and negations are given in [20].

In [14], the following forms of De Morgan's Laws

$$\neg(\neg x \vee \neg y) = x \wedge y \quad \text{and} \quad \neg(\neg x \wedge \neg y) = x \vee y$$

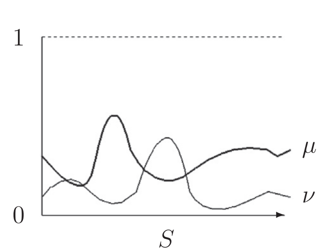

Fig. 1. First geometrical interpretation of an intuitionistic fuzzy set.

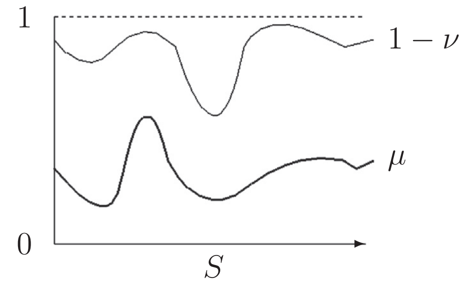

Fig. 2. First geometrical interpretation of an intuitionistic fuzzy set (a modified form).

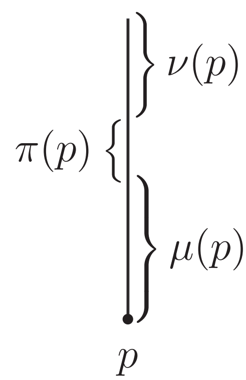

Fig. 3. First geometrical interpretation of an element of the intuitionistic fuzzy set.

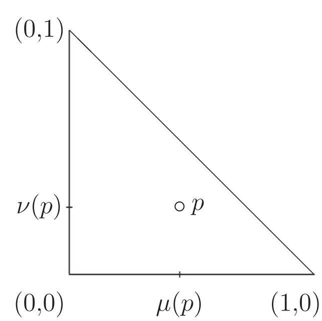

Fig. 4. Second geometrical interpretation of an intuitionistic fuzzy set.

and

$$\neg(\neg x \vee \neg y) = \neg\neg x \wedge \neg\neg y \quad \text{and} \quad \neg(\neg x \wedge \neg y) = \neg\neg x \vee \neg\neg y,$$

and the following forms of the Law for Excluded Third

$$x \vee \neg x \quad \text{and} \quad \neg\neg x \vee \neg x$$

are discussed.

We can check, for example, that the standard forms of De Morgan's Laws

$$\neg x \vee \neg y = \neg(x \wedge y) \quad \text{and} \quad \neg x \wedge \neg y = \neg(x \vee y)$$

and the first two from the above forms are valid for negations $\neg_1 \langle a, b \rangle = \langle b, a \rangle$ and $\neg_2 \langle a, b \rangle = \langle \overline{\text{sg}}(a), \text{sg}(a) \rangle$, while only negation $\neg_2$ satisfies the two above forms and does not satisfy the standard De Morgan's Laws.

The most important problem is related to T- and S-norms, defined for intuitionistic fuzzy case. In the present moment, they satisfy the standard De Morgan Laws and therefore, they are based on classical negation $\neg_1$. An open problem for future research is to develop a new theory of T- and S-norms for intuitionistic fuzzy case, which are based on the modified forms of De Morgan Laws.