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A survey of approaches to decision making with intuitionistic fuzzy preference relations

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ABSTRACT

Intuitionistic fuzzy preference relations (IFPRs) have attracted more and more scholars' attentions in recent years due to their efficiency in representing experts' imprecise cognitions. With IFPRs, people can express their opinions over different pairs of alternatives from positive, negative and hesitative points of view. This paper presents a comprehensive survey on decision making with IFPRs with the aim of providing a clear perspective on the originality, the consistency, the prioritization, and the consensus of IFPRs. Finally, some directions for future research are pointed out.

1. Introduction

Decision making takes place everywhere and every time in our daily life. Before making any decision, the first thing we should do is to collect sufficient information related to the considered problem. In many cases, the information is determined according to the decision makers' opinions or the experts' assessments. Therefore, how to describe the group members' opinions is very important and it influences the final decision result directly as they often are only able to express their opinions roughly and subjectively. Generally speaking, there are mainly three ways in which the decision makers or the experts can express their opinions: preference orderings, utility values, and preference relations. Preference orderings are a collection of natural numbers which are a permutation of $(1, 2, \ldots, n)$ used by the experts for showing the order positions of a set of alternatives in sequence [64]. Utility values are a series of exact real numbers taken from a closed unit interval [0,1] to indicate the preferences of a decision maker towards different outcomes [8]. Preference relation is constructed via pairwise comparisons over the alternatives, and each value in it indicates the preference degree or intensity of one alternative over another [37]. Comparing these three representation tools, the preference orderings are oversimplified because they contain little information about the experts' preferences, which makes it inconvenient or impossible for further investigation especially when a group of experts cannot reach a mutually agreeable result. The utility values of the alternatives are sometimes very difficult to be determined. In addition, as pointed out by Winkler [54], *utility theory is lacking as a descriptive theory of how people actually behave if left their own devices*. Such a descriptive theory will never be prescriptively appealing [53]. However, the preference relations do not have these limitations. It can express an expert's judgments subjectively according to his/her cognition. With the preference relations, there is no need for the experts to determine the crisp utility values of alternatives over each criterion. Basically, there are two types of preference relations, which are the multiplicative preference relations and the fuzzy preference relations. Based on the multiplicative preference relations, the famous analytic hierarchy process (AHP) method was proposed [37]. In classical AHP method, the 1–9 scale is used as fundamental scale to represent judgments in forms of pairwise comparisons and thus a multiplicative preference relation should be constructed. As all the judgments in a multiplicative preference relation are crisp values which are hard to be exactly furnished in many complex and uncertain cases, a fuzzy preference relation (FPR) was then introduced [36]. The FPR uses a number from the interval [0,1] to characterize the degree of certainty in the preference between a pair of alternatives. A FPR may arise when each expert is not unambiguously certain as to $A_i > A_j$, or different experts have different opinions as to $A_i > A_j$ in which case a fraction of the number of experts having voted for $A_i > A_j$ is taken as a degree of $A_i > A_j$ [50]. Tanino [50] firstly gave the formal definition of FPR as a fuzzy binary relation matrix satisfying reciprocal condition and max–min transitivity. Although the

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multiplicative preference relations and the fuzzy preference relations have attracted significant attention of many scholars due to their efficiency, there are still some weaknesses on the fuzzy preference relations and the multiplicative preference relations. Both of them consider only the preference degrees or intensities of one alternative against another. In many cases, if some of the experts are not very familiar with the decision making problem or there exists some incomplete information about some of the alternatives, it might be very difficult for the experts to determine such preference degrees.

In 1983, Atanassov [1] extended the traditional fuzzy set into the intuitionistic fuzzy set by considering the degrees of hesitancy. With this type of representation technique, when comparing pairs of alternatives, the experts can express their imprecise cognitions from the positive, negative and hesitative points of view and thus construct an intuitionistic fuzzy preference relation (IFPR) [48,66]. Xu [66] defined an IFPR $\tilde{R}$ as a preference structure, whose elements are intuitionistic fuzzy numbers (IFNs), denoted as $\tilde{r}_{ij} = (\mu_{ij}, \nu_{ij}, \pi_{ij})$ with $\mu_{ij},\ \nu_{ij} \in [0,1], \mu_{ij} + \nu_{ij} \leqslant 1, \mu_{ij} = \nu_{ji}, \mu_{ii} = \nu_{ii} = 0.5$, for all $i,j = 1,2,\ldots,n, l = 1,2,\ldots,s$. μ_{ij} means the preference degree of the alternative A_i to A_j; ν_{ij} indicates the non-preference degree of the alternative A_i to A_j, and $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$ is interpreted as an indeterminacy degree or a hesitancy degree. As an IFPR can express the opinions of an expert in terms of "preferred", "not preferred", and "indeterminate" aspects, it is more comprehensive and flexible than the fuzzy preference relation and the multiplicative preference relation in expressing an expert's preferences. Montero and Gomez [34] viewed the preference modeling as a classification problem, which is deeply related to intuitionistic fuzzy sets.

The IFPR has been implemented into many different aspects. After giving the formal definition of IFPR, Xu [66] applied it into the process of assessing a set of agroecological regions in Hubei Province, China. In this example, Hubei province was divided into seven agroecological regions and three experts were asked to prioritize them with respect to their comprehensive functions. After establishing the integral processes of intuitionistic fuzzy AHP (IFAHP) method, Xu and Liao [67] then applied this method into the global supplier development problem in which the experts provided their assessments in terms of IFPRs. Xu [62] also used the IFPR to aid the customer to buy a refrigerator. In this example, the evaluator compared each pair of refrigerators and constructed an IFPR to represent the opinions of the evaluator over these refrigerators. The linear programming method were utilized to produce the priorities of these refrigerators. Based on the additive consistency as well as the least squares optimization method and the goal programming method, Gong et al. [23] used the IFPR to analyze and assess the industry meteorological service for China Meteorological Administration. On the other hand, according to the multiplicative consistency of IFPR, Gong et al. [24] took the IFPR as a tool to represent the evaluation information of house buyer and then used the goal programming method to help the buyer to rank the candidate houses. Wang [52] used the IFPR to help a customer to select a new vehicle to buy. He also showed how to use the IFPR to select the international exchange doctoral students. Liao and Xu [26] proposed an automatic procedure to repair the inconsistent IFPR and developed an algorithm to aid the decision makers to analyze the performance of three types of motorcycles. After introducing a new definition of multiplicative consistency for the IFPR, Liao and Xu [29] implemented the IFPR into the process of selecting a flexible manufacturing system (FMS). Afterward, Liao and Xu [30] developed some fractional models for group decision making with IFPRs and then implemented these methodologies into a group decision making problem concerning the evaluation and ranking of the main factors of electronic learning. To show the efficiency of the error-analysis-based method, Xu [61] applied the IFPR into the supply chain management problem to determine the importance of the factors which can influence the cooperation among enterprises. Recently, Liao et al. [33] proposed the framework of group decision making with IFPRs and applied the proposed decision making process to select outstanding PhD students for China Scholarship Council. In the process of group decision making with IFPRs, assuming that each IFPR is irreflexive, asymmetric and transitive, Dimitrov [18] proved that the aggregation rule [17] maps also each profile of such preferences into an irreflexive, asymmetric and transitive intuitionistic fuzzy collective preference relation.

In order to better understand the state of the art of IFPRs, this paper provides an extensive and intensive overview on IFPRs, including its originality of concept, transitivity and consistency, priority methods and consensus measures. Based on these objectives, the remainder of this paper is set out as follows: Section 2 reviews the originality of IFPR. After recalling the transitivity of IFPR, Section 3 discusses the state of the art of the consistency of an IFPR, including the different forms of additive consistency and the different forms of multiplicative consistency. In Section 4, a survey concerning the priority methods is given. Section 5 mainly addresses different types of consensus measures for group decision making with IFPRs. The paper ends with some concluding remarks in Section 6.

2. Preference relations and intuitionistic fuzzy preference relation

In analytic hierarchy process (AHP), Saaty [37] decomposed a complex multi-criteria decision making (MCDM) problem into a multi-level hierarchic structure of objectives, criteria, sub-criteria and alternatives, and then provided a fundamental scale of relative magnitudes expressed in dominance units to represent judgments in the form of pairwise comparisons. The fundamental scale expresses relative importance of the elements in a level with respect to the elements in the level immediately above it.

Definition 1 [37]. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a finite set of alternatives and $C = \{C_1, C_2, \ldots, C_m\}$ be a set of criteria to compare the alternatives. A fundamental scale for the criteria $C_j \in C$ $(j = 1,2,\ldots,m)$ is a mapping P_{C_j}, which assigns to every pair $(A_i, A_k) \in A \times A$ a positive real number $P_{C_j}(A_i, A_k) = p_{ik}$ that denotes the relative intensity with which an individual perceives the criterion $C_j \in C$ in an element $A_i \in A$ in relation to the other $A_k \in A$.

In addition, Saaty further developed the 1–9 scale to describe the preferences between alternatives as being either equally, moderately, strongly, very strongly or extremely preferred. These preferences are translated into pairwise weights of one, three, five, seven or nine, respectively, with two, four, six, eight as the intermediate values (see Table 1 for more details). The 1–9 scale satisfies the reciprocal condition, i.e., the intensity of preference of A_i over A_k is inversely related to the intensity of preference of A_k over A_i, that is,

$$p_{ik} = 1/p_{ki}, \quad \forall A_i, A_k \in A, \ C_j \in C, \ j = 1,2,\ldots,m \tag{1}$$

With the 1–9 scale, in general, Saaty pointed out that: $A_i \succ_{C_j} A_k$ if and only if $p_{ik} > 1$ where the binary relation "$\succ_{C_j}$" represents "be preferred to" according to the criterion C_j; $A_i \sim_{C_j} A_k$ if and only if $p_{ik} = 1$ where the binary relation "$\sim_{C_j}$" represents "be indifferent to" according to the criterion C_j.

If all the pairwise judgments determined by the experts with 1–9 scale are stored in an $n \times n$ matrix $A = (a_{ik})_{n \times n}$, then a multiplicative preference relation is constructed: