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Designing granular fuzzy models: A hierarchical approach to fuzzy modeling

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ABSTRACT

In this study, we elaborate on a distributed fuzzy modeling and ensuing granular fuzzy modeling. Such modeling is realized in the presence of separate and locally available data while the ensuing fuzzy rule-based models constructed on their basis are regarded as individual sources of knowledge. In virtue of an inherent diversity of these sources (models) and in an attempt to quantify it, a global model being formed at the higher level of hierarchy is becoming more abstract than those at the lower level is referred to as a *granular* fuzzy model. An essential concept of this class of models is introduced and their enhanced functionality is studied. Furthermore, we show interesting linkages of these models with type-2 fuzzy models studied in the literature. We highlight a number of arguments motivating a need and justifiable relevance of higher type of information granules. A detailed discussion on fuzzy rule-based models exhibiting an interesting aspect of an incremental format of the rules (whose rules capture an incremental description of input–output relationships formed with respect to some simple reference model (say, constant or linear) is presented. The design practice of the models is elaborated on by highlighting in this context the use of augmented fuzzy clustering. The construction of a granular fuzzy model is guided by the principle of justifiable granularity using which we show how granular parameters of the models are formed. The performance of the model is quantified with respect to the two criteria, namely coverage of experimental data and specificity of granular results. Experimental studies are reported for both synthetic and publicly available data sets.

1. Introduction

In fuzzy modeling coming today with an impressive plethora of design methodologies and algorithms we are almost exclusively concerned with a construction of individual fuzzy models built on a basis of single data sets. While these pursuits address a variety of needs, there is an evident limitation at the methodological side. In a number of practical scenarios we encounter a series of local data [1] describing a certain view at the phenomenon, giving rise to some local fuzzy models and subsequently offering a certain *local* and limited view at the system under study. Forming a single fuzzy model of a *global* character implies its localization at the higher level of abstraction. Not venturing into a possibly large number of modeling alternatives that could be worth exploring here, first of all it is beneficial to become cognizant of some general and intuitively compelling nature of global models. To be reflective of the diversity of local models (treated as sources of knowledge) a global model should be able to capture this variety and quantify it. Intuitively, we may envision that such global model should be established at the higher level of abstraction than the original local fuzzy models. This strongly advocates a new concept of generalized models, which can be descriptively referred to as *granular fuzzy models*. In comparison with fuzzy models whose parameters are described in terms of fuzzy sets with numeric membership functions, granular fuzzy models are endowed with granular fuzzy sets of parameters. By granular fuzzy sets we mean fuzzy sets whose membership characterization is realized in terms of information granules. Information granularity plays here a pivotal role in reflecting and prudently quantifying the diversity of the local fuzzy models. The ultimate objective of this study is to establish a general development environment and deliver a strong motivation behind an emergence of the class of granular fuzzy models.

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The key design scenario investigated here is outlined as follows: given a collection of local fuzzy models, construct a granular fuzzy model whose parameters are granular fuzzy sets.

It is instructive to cast this study in a certain general setting by alluding to the main results obtained in the framework of Granular Computing and type-2 fuzzy modeling, in particular [2–7] and some recent developments reported in [8–12]. There are two main categories of development strategies and ensuing classes of models, refer to Fig. 1.

(i) Granular aggregation of results produced by several local sources of knowledge (models) For a certain input $\boldsymbol{x}$, local models produce (typically) different outcomes which are aggregated yielding granular rather than numeric output (being encountered in commonly considered aggregation mechanisms). The produced information granule is formed with the aid of the principle of justifiable granularity. It is worth noting that information granules arise at the level of aggregation of results (output of the models) and in this sense we are not seeing here emergence of any new granular models, (ii) granular models augmenting existing numeric models. It is apparent that there are no *ideal* numeric models. A granular model is built on a basis of the existing model by making its parameters granular and in this way delivering a required "coverage" of the experimental data [7] and quantifying the performance of the model. In the design process, we take advantage of information granularity which is sought as an essential design asset whose allocation (distribution) across the parameters of the model can be optimized following a certain coverage criterion, see [7,13]. (iii) in contrast to (ii) where a single model is studied, the buildup of the granular fuzzy model in (iii) is realized for a collection of local fuzzy models.

There are some commonalities between the granular constructs presented here and type-2 fuzzy models and modeling [5,6] however there are also striking differences. This study delivers a significant originality and there is a strong motivation behind the discussion presented here. First, granular fuzzy sets bring forward a unified view at the granular nature of system modeling. Likewise they try to establish a unified view at the granular nature of the construct where type-2, interval-valued fuzzy sets or probabilistic sets are regarded as selected cases of granular fuzzy models. Second, granular fuzzy models are built on a basis of locally available fuzzy models and their design directly dwells on the use of the local constructs and is fully supported by exploiting the methodology of Granular Computing [14], especially, the principle of justifiable granularity [15] serving as a vehicle behind a formation of information granules. This comes in a sharp contrast with a rationale and a way of constructing of type-2 fuzzy models in which such models are built from scratch (which quite often comes with quite excessive optimization effort). The underlying motivation is also quite different: the main argument is that of the higher flexibility of type-2 fuzzy sets, which become beneficial to tackle uncertainty of modeled systems. Ironically, this feature is not fully exploited as at the end a type-reduction (and decoding) makes this potentially important capability not fully exploited.

The study is structured in a bottom up fashion. We start (Section 2) by introducing a generic version of local fuzzy models of rule-based character, which is general enough, builds directly on the structure of data revealed through fuzzy clustering. As clustering is of relational (direction-free) nature, we augment the clustering process by some additional flexibility to cope with the directionality aspect that helps us tell apart input and output variables of the model. In Section 3, we present some experimental results showing the performance of the model and discussing an impact of the essential parameters on its performance. A generalization of the fuzzy model in a form of its granular counterpart is presented in Section 4; here we elaborate on both the structure of the model and a construction of the granular parameters realized with the help of the principle of justifiable granularity and involving parameters of the local fuzzy models. The assessment of the quality of the model is discussed in Section 5 while experimental results are covered in Section 6.

2. A structure of the fuzzy model

We are concerned with the fuzzy model whose structure (in essence being a certain rule-based architecture) is built directly by discovering and exploiting the structure of the data being determined with the use of an augmented model-oriented fuzzy clustering. In the construction of the model we use a collection of data coming in the form of N input–output pairs ($\boldsymbol{x}_k$, $target_k$), $k = 1, 2, .. , N$ where $\mathbf{x}_k \in \boldsymbol{R}^n$, $target_k \in \boldsymbol{R}$.

2.1. The topology of the fuzzy model

The model composed of c rules are given in the following form

$$y = \bar{y} + \sum_{i=1}^{c} A_i(\mathbf{x})\bar{\varepsilon}_i \tag{1}$$

where

$$\bar{y} = \frac{1}{N}\sum_{k=1}^{N} target_k \tag{2}$$

is an average of the output data and $\bar{\varepsilon}_i$ is an average error (viz. the differences between the data and the results produced by the generic model $\bar{y}$) produced by the global model for the inputs being localized within the neighborhood of A_i. A_i is a fuzzy set defined in the n-dimensional input space and determined by running fuzzy clustering (Fuzzy C-Means, FCM). The membership functions of fuzzy sets $A_1, A_2, \ldots, A_c$ read as follows

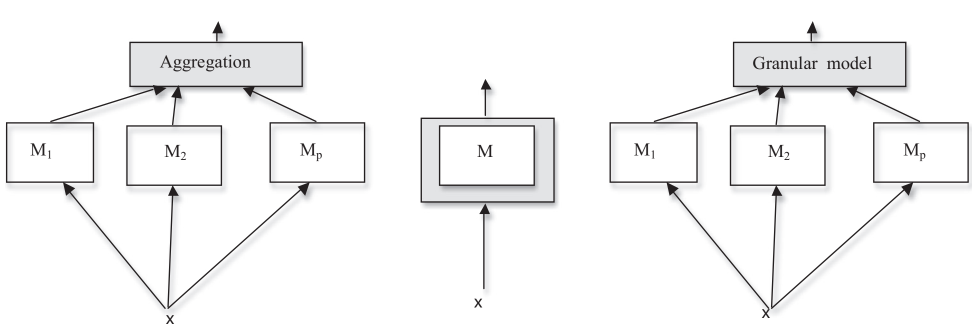

Fig. 1. Categories of granular constructs: (i) granular aggregation of results produced by fuzzy models $M_1, M_2, \ldots, M_p$, (ii) augmented granular model, and (iii) granular model formed at the higher level of hierarchy. Shaded symbols are used to visualize the localization of information granules.