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Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets

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ABSTRACT

The hesitant fuzzy linguistic term set (HFLTS) is a new and flexible tool in representing hesitant qualitative information in decision making. Correlation measures and correlation coefficients have been applied widely in many research domains and practical fields. This paper focuses on the correlation measures and correlation coefficients of HFLTSs. To start the investigation, the definition of HFLTS is improved and the concept of hesitant fuzzy linguistic element (HFLE) is introduced. Motivated by the idea of traditional correlation coefficients of fuzzy sets, intuitionistic fuzzy sets and hesitant fuzzy sets, several different types of correlation coefficients for HFLTSs are proposed. The prominent properties of these correlation coefficients are then investigated. In addition, considering that different HFLEs may have different weights, the weighted correlation coefficients and ordered weighted correlation coefficients are further investigated. Finally, an application example concerning the traditional Chinese medical diagnosis is given to illustrate the applicability and validation of the proposed correlation coefficients of HFLTSs in the process of qualitative decision making.

1. Introduction

Fuzzy knowledge based systems are based on the fact that experts usually rely on **common sense** from their domain knowledge when they solve problems. In addition, they also use ambiguous terms to express their cognition [1]. A simple example coming from the power system is like this: an expert who is in charge of the generator might say, "*Though the power transformer is **slightly** overloaded, I can keep this load for **a while***." In such a situation, it is impossible for the expert or his/her audiences to describe the term such as "slightly" or "a while" in crisp numerical value, but all of the audiences can understand what does that mean. In many real-life qualitative decision making problems, it is very common and straightforward for experts to express their opinions in terms of linguistic terms, such as "fast" speed, "high" price, "low" temperature, and "good" performance. Although linguistic terms are very close to human's cognitive process, computing with such linguistic terms is not easy. In 1975, Zadeh [2] proposed the fuzzy linguistic approach, which uses linguistic variables, whose values are not numbers but words or sentences in a natural or artificial language, to represent qualitative information of a person. In spite of being less precise than a number, the linguistic variable enhances the feasibility, flexibility and reliability of decision models and provides good results in different fields [3].

Nevertheless, as the fuzzy linguistic approach uses only one linguistic term to represent the value of a linguistic variable, it sometimes may not reflect exactly what the experts mean. In many cases with high degree of uncertainty, the experts might hesitant among several linguistic terms and need richer linguistic expressions to represent their opinions. For example, when evaluating the performance of a company, an expert may say "*it is **not too bad***"; another expert may say "its performance is ***between medium and high***." The traditional fuzzy linguistic approach cannot represent such comprehensive linguistic expressions. Recently, Rodríguez et al. [4] proposed a new proposal to improve the elicitation of linguistic information by using hesitant fuzzy linguistic term set (HFLTS) and context-free grammars. The HFLTS increases the flexibility and capability of elicitation of linguistic information by means of linguistic expressions. The context-free grammars fix the rules for the experts to build such flexible linguistic expressions, which can be transformed into HFLTS. With the use of HFLTS, the experts can provide their assessments by means of several linguistic terms or comparative linguistic expressions.

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Since the HFLTS provides a new and more powerful technique to represent experts' qualitative judgments, it has attracted more and more scholars' attention. Rodríguez et al. [4] introduced the concept of HFLTS, and investigated some basic operations and properties of HFLTS. A multi-criteria linguistic decision making model with linguistic expressions based on comparative terms was also given by them. Liao et al. [5] introduced a sort of distance and similarity measures for HFLTSs, based on which, a satisfactory-based decision making method was given for multi-criteria decision making (MCDM) under hesitant fuzzy linguistic circumstance. Wei et al. [6] developed some comparison methods and studied the aggregation theory for HFLTS. Chen and Hong [7] presented a new method for multi-criteria linguistic decision based on HFLTSs using the pessimistic attitude and the optimistic attitude of the decision maker. By means of a fuzzy envelope, Liu and Rodríguez [8] proposed a new representation of the HFLTS, which can be used to carry out the computing with words processes. Rodríguez et al. [9] also gave a fuzzy representation for the semantics of HFLTSs. Zhu and Xu [10] defined the hesitant fuzzy linguistic preference relation (HFLPR) and investigated its consistency. Liu et al. [11] investigated the additive consistency of HFLPR. Based on HFLTS and context-free grammars, Rodríguez et al. [12] developed a new linguistic group decision making model which deals with comparative linguistic expressions that are similar to those used by the experts in real-world decision making problems. Beg and Rashid [13] proposed a TOPSIS-based method for MCDM in which the opinion of the experts is represented by HFLTS. In order to handle hesitant fuzzy linguistic MCDM where some criteria conflict with each other, recently, Liao et al. [14] gave a step by step procedure of HFL-VIKOR method and validated it via some numerical examples.

All these above literatures show that HFLTS is a hot topic in both theoretical and practical fields. As HFLTS has been proposed for just a few years, much work needs to be done to enrich the framework of HFLTS theory. As it is well known, correlation measure is one of the most widely used indices in varying fields [15–28]. However, up to now, as far as we know, there is no research on the correlation measure of HFLTSs. Hence, in this paper, we focus on this issue and propose several important correlation measures and correlation coefficients for HFLTSs. To do so, the remainder of this paper is organized as follows: Section 2 gives some basic knowledge on fuzzy linguistic approach and HFLTS. The definition of HFLTS is improved and the HFLE is introduced. A short review on the correlation measures over fuzzy sets and its extensions is also given in this section. Section 3 proposes different forms of correlation measures and correlation coefficients for HFLTSs. The properties of these correlation coefficients are investigated in this section as well. In Section 4, the weighted correlation coefficients and ordered weighted correlation coefficients are investigated. An application example concerning the traditional Chinese medical diagnosis is given in Section 5 to show the applicability and validation of these correlation coefficients of HFLTSs. The paper ends with some concluding remarks in Section 6.

2. Preliminaries

2.1. Fuzzy linguistic approach

The fuzzy linguistic approach [2] was proposed to model linguistic information proposed by experts. In such an approach, the experts' opinions are taken as the values of a linguistic variable which is established by a linguistic descriptors and its semantics. Many different models were proposed to represent and calculate the values of a linguistic variable, such as the semantic model [3], the virtual linguist model [29], the 2-tuple linguistic model [30], and the proportional 2-tuple linguistic model [31]. The virtual linguist model is easy and straightforward, and it has been used by many scholars. A subscript-symmetric additive linguistic term set [32] is shown as:

$$S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\} \quad (1)$$

where the mid linguistic label s_0 represents an assessment of "indifference", and the rest of them are placed symmetrically around it. In particular, $s_{-\tau}$ and s_τ are the lower and upper bounds of linguistic labels used by experts in practical applications, τ is a positive integer, and S satisfies the following conditions:

(1) If $\alpha > \beta$, then $s_\alpha > s_\beta$;
(2) The negation operator is defined: $\mathrm{neg}(s_\alpha) = s_{-\alpha}$, especially, $\mathrm{neg}(s_0) = s_0$.

Since the linguistic term set S is a discrete linguistic term set, it is not convenient for calculating and analyzing. In order to preserve all given linguistic information, Xu [29] extended the discrete linguistic term set into continuous linguistic term set $\bar{S} = \{s_\alpha | \alpha \in [-q, q]\}$, where $q (q > \tau)$ is a sufficiently large positive integer. In general, the linguistic term $s_\alpha (s_\alpha \in S)$ is determined by the experts, while the extended linguistic term (named virtual linguistic term), $\bar{s}_\alpha (\bar{s}_\alpha \in \bar{S})$, only appears in computation process.

For any two linguistic terms $s_\alpha, s_\beta \in \bar{S}$ and $\lambda, \lambda_1, \lambda_2 \in [0, 1]$, the following operational laws were introduced [29]:

(1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;
(2) $\lambda s_\alpha = s_{\lambda\alpha}$;
(3) $(\lambda_1 + \lambda_2) s_\alpha = \lambda_1 s_\alpha \oplus \lambda_2 s_\alpha$;
(4) $\lambda (s_\alpha \oplus s_\beta) = \lambda s_\alpha \oplus \lambda s_\beta$.

2.2. Hesitant fuzzy linguistic term set

In quantitative settings, when an expert considers several values to determine the membership degree of an element to a set, the concept of hesitant fuzzy set (HFS) was introduced [33,34]. As for qualitative circumstances, when establishing the value of a linguistic variable, several linguistic terms may be elicited. Thus, motivated by the idea of HFS, Rodríguez et al. [4] introduced the concept of HFLTS.

Definition 1 [4]. Let $S = \{s_0, \ldots, s_\tau\}$ be a linguistic term set. A hesitant fuzzy linguistic term set (HFLTS), H_S, is an ordered finite subset of the consecutive linguistic terms of S.

The HFLTS can be used to elicit several linguistic values for a linguistic variable, but it is not similar to human way of thinking and reasoning. In order to make it more applicable, Rodríguez et al. [4] proposed a context-free grammar G_H to generate simple but elaborated linguistic expressions ll that are similar to the human's expressions. The expressions ll generated by the context-free grammar G_H may be either single valued linguistic terms or linguistic expressions. The transformation function E_{G_H} can be used to transform the expressions ll that are produced by G_H into HFLTS H_S (for more details, please refer to Refs. [4,12,14]). The way to obtain a HFLTS can be shown as Fig. 1.

It is noted that, regarding to the linguistic term set $S = \{s_0, \ldots, s_\tau\}$ given in Definition 1, when its subscripts are not symmetric, some problems will arise. For example, for a linguistic term set $S = \{s_0 = none, s_1 = very\ low, s_2 = low, s_3 = medium, s_4 = high, s_5 = very\ high, s_6 = perfect\}$, according to the operational law, we have $s_2 \oplus s_3 = s_5$, which means, the aggregated result of linguistic terms "*low*" and "*medium*" is "*very high*". This is not coincident with our intuition. (for more details, see Refs. [5,10,14]). To overcome these problems, Liao et al. [5] replaced the linguistic