Knowledge-Based Systems 71 (2014) 187–200

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Multiplicative consistency of intuitionistic reciprocal preference relations and its application to missing values estimation and consensus building

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ARTICLE INFO

Article history:
Received 7 March 2014
Received in revised form 28 July 2014
Accepted 29 July 2014
Available online 7 August 2014

Keywords:
Multiplicative consistency
Intuitionistic reciprocal preference relations
Missing preferences
Group decision and negotiation
Consensus

ABSTRACT

The mathematical modelling and representation of Tanino's multiplicative transitivity property to the case of intuitionistic reciprocal preference relations (IRPRs) is derived via Zadeh's extension principle and the representation theorem of fuzzy sets. This result guarantees the correct generalisation of the multiplicative transitivity property of reciprocal preference relations (RPRs), and it allows the multiplicative consistency (MC) property of IRPRs to be defined. The MC property used in decision making problems is threefold: (1) to develop a consistency based procedure to estimate missing values in IRPRs using an indirect chain of alternatives; (2) to quantify the consistency index (CI) of preferences provided by experts; and (3) to build a novel consistency based induced ordered weighted averaging (MC-IOWA) operator that associates a higher contribution in the aggregated value to the more consistent information. These three uses are implemented in developing a consensus model for GDM problems with incomplete IRPRs in which the level of agreement between the experts' individual IRPRs and the collective IRPR, which is referred here as the proximity index (PI), is combined with the CI to design a feedback mechanism to support experts to change some of their preference values using simple advice rules that aim at increasing the level of agreement while, at the same time, keeping a high degree of consistency. In the presence of missing information, the feedback mechanism implements the consistency based procedure to produce appropriate estimate values of the missing ones based on the given information provided by the experts. Under the assumption of constant CI values, the feedback mechanism is proved to converge to unanimous consensus when all experts are provided with recommendations and these are fully implemented. Additionally, visual representation of experts' consensus position within the group before and after implementing their feedback advice is also provided, which help an expert to revisit his evaluations and make changes if considered appropriate to achieve a higher consensus level. Finally, an IRPR fuzzy majority based quantifier-guided non-dominance degree based prioritisation method using the associated score reciprocal preference relation is proposed to obtain the final solution of consensus.

1. Introduction

Group decision making (GDM) consists of multiple individuals interacting to reach a decision based on the information they provide. Given two alternatives of a finite set of all potentially available, X, an expert either prefers one to the other or is indifferent between them. Obviously, there is another possibility, that of an expert being unable to compare them.

There exist two main mathematical frameworks based on the concept of preference relation. In the first one a preference relation is defined for each one of the above three possible preference states, which is usually referred to as a preference structure on the set of alternatives [33]. The second one integrates the three possible preference states into a single preference relation [5]. Further to this, in each case two different representations could be adopted: the use of binary (crisp) preference relations or the use of [0,1]-valued (fuzzy) preference relations. Reciprocal [0,1]-valued relations $(P=(p_{ij});\ \forall i,j:\ 0\leqslant p_{ij}\leqslant 1,\ p_{ij}+p_{ji}=1)$ are frequently used in fuzzy set theory for representing intensities of preferences [5,10,31,36]. These types of relations will be referred to as simply reciprocal preference relations, and are the ones used

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http://dx.doi.org/10.1016/j.knosys.2014.07.024
0950-7051/

in this paper. In probabilistic choice theory, reciprocal preference relations describe the binary preferences subsets of two-alternatives of X, and are known with the name 'probabilistic binary preference relations' [30]. Reciprocal preference relations can be seen as a particular case of (weakly) complete fuzzy preference relations [22], i.e. fuzzy preference relations satisfying $p_{ij} + p_{ji} \geqslant 1 \ \forall i, j$.

An exhaustive survey of the second type of preference relations mentioned above is given in [43], with the main relations used in the literature to capture uncertainty of information being: the multiplicative preference relation (MPR), the reciprocal preference relation (RPR), the linguistic preference relation (LPR), and the intuitionistic reciprocal preference relation (IRPR). The last one is based on Atanassov's intuitionistic fuzzy set (IFS) introduced in [3]. An IFS is based on the use of membership degree, non-membership degree and hesitation index to model experts' subjective preferences. Indeed, there may be some real-life decision making cases where a decision maker (DM) may not be able to accurately express his/her preferences for some or all of the alternatives because he/she is not completely confident or presents some hesitation [17], making they use of intuitionistic fuzzy values very suitable to model and represent the DM's preference rather than other type of preference representation formats [44]. Intuitionistic fuzzy values have been used for example in Iwate (Japan) by Professor Fujita and his research laboratory to represent and model medical doctor responses in medical diagnosis as part of the *mental cloning* "used to mirror a person cognitive behaviour into a model that interacts with human users" [6] on building the virtual doctor system (VDS) for medical applications [7–9]. Recently, the use of IRPRs in decision making in uncertain environments has attracted the attention of many researchers [23,24,39].

An issue to address in GDM problems with IRPRs is the lack of information, a problem extensively studied in the case of RPRs [14,20]. In this context, consistency based methods to 'estimate' the missing values from known ones have been proposed in [1,2,27], which later were extended to the GDM framework [28,29]. However, modelling and evaluating the consistency of IRPRs has not yet been solved.

'Some individual opinions can be considered more consistent than other individual opinions', which was used by Cutello and Montero in [16] to claim that rationality of individuals can be considered a fuzzy concept, where they addressed the problem of modelling rationality of individuals based only on their opinions over a finite and fixed set of alternatives expressed using complete fuzzy preference relations. They characterised fuzzy rationality measures which are explicitly consistent by establishing a collection of conditions to satisfy. Explicit consistency is defined in [16] as 'absence of explicit contradictions, i.e. statements of type $(P \wedge \neg P)$.'

Consistency of RPRs has been modelled using the notion of transitivity, in the sense that if alternative x_i is preferred to alternative $x_j (p_{ij} \geqslant 0.5)$ and this one to $x_k (p_{jk} \geqslant 0.5)$ then alternative x_i should be preferred to $x_k (p_{ik} \geqslant 0.5)$. This transitivity notion is normally referred to as *weak stochastic transitivity* [30,36]. However, the implementation of the intensity of preference in modelling consistency of RPRs has been proposed in many different ways [27]. Tanino in [36] proposed the additive transitivity property, which is although equivalent to Saaty's consistency property for MPRs [35], is in conflict with the $[0,1]$ scale used for providing the preference values and therefore it is an inappropriate property to model consistency of reciprocal preference relations [12]. In [36], Tanino also proposed a multiplicative transitivity property for RPRs, which has been characterised to be the most appropriate one for modelling cardinal consistency of RPRs [12]. Recall that RPRs are particular cases of IRPRs, and therefore the same previous claim can be applied to them. Thus, the first objective in this paper is to formalise the multiplicative transitivity property for IRPRs. Once this is achieved, a methodology will be developed to (1) quantify the level of consistency or consistency index (CI) of an IRPR, and (2) estimate missing values of incomplete IRPRs. Because consistent information is considered more relevant or important than inconsistent information, an intuitionistic aggregation operator that associates higher weights with more consistent information will be developed. In other words, a new MC induced ordered weighted averaging (MC-IOWA) operator is defined and proposed to compute the collective IRPR.

As aforementioned, consistency is linked to rationality of individuals, and therefore it has been considered as a reasonable criteria to guide consensus reaching processes [11,37]. On the other hand, similarity interpreted as a measure of general or widespread agreement, based on the use of a metric or distance function, is usually regarded as a criteria to use in measuring consensus [15,18,40,50]. By combining both consistency and similarity functions, Herrera-Viedma et al. [29] developed a feedback mechanism to provide advice to experts in order to increase the consensus level of the group. Chiclana et al. [13] and Wu and Xu [42] designed a two stage model with a first stage aiming to reach acceptable consistency level while the second one was used to achieve a predefined consensus level. Different from the above consensus models, Dong et al. [19] investigated a minimum cost optimisation model to reach acceptable consensus in which the individual consistency and consensus level are used as two limiting conditions simultaneously. Obviously, using these two criteria simultaneously in consensus process seems to be more reliable than just one criteria.

Therefore, the second objective of this paper is to investigate a consensus reaching process for IRPRs that combines these two criteria. Approaches to model consensus in GDM problems with IRPRs are already available in literature [34,44]. However, there exist two main limitations in these consensus models: (i) they did not take into account of the consistency of individual preference; and (ii) they are static in nature because, when there is not enough consensus, they do not include any type of feedback process to advise the experts on how to change their preferences to increase consensus. To overcome these limitations, and inspired by the work of Herrera-Viedma et al. [29], both consistency and consensus levels will be implemented in the design of a feedback mechanism to support experts to change some of their preference values using simple advice rules that aim at increasing the level of agreement while, at the same time, keeping a high degree of consistency. In the presence of missing information, the feedback mechanism implements the consistency based procedure to produce appropriate estimate values of the missing ones based on the given information provided by the experts. Under the assumption of constant CI values, the feedback mechanism is proved to converge to unanimous consensus when all experts are provided with recommendations and these are fully implemented. Additionally, visual representation of experts' consensus position within the group before and after implementing their feedback advice is also provided, which helps an expert to revisit his evaluations and make changes if considered appropriate to achieve a higher consensus level. Finally, an IRPR fuzzy majority based quantifier-guided non-dominance degree based prioritisation method using the associated score reciprocal preference relation is proposed to obtain the final solution of consensus.

The rest of paper is set out as follows: Section 2 presents the formal approach to extend the mathematical expression of multiplicative transitivity property from RPR to IRPR, as well as the definition of multiplicative consistent IRPR and consistency indexes of IRPRs. In Section 3, a multiplicative consistency based method to estimate missing values of IRPRs is proposed. The consensus model for GDM with incomplete IRPRs is covered in Section 4, with special attention paid to the design of the consistency-consensus