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Rule-preserved object compression in formal decision contexts using concept lattices

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A B S T R A C T

Rule acquisition is one of the main purposes in the analysis of formal decision contexts. In general, given a formal decision context, some of its objects may not be essential to the rule acquisition. This study investigates the issue of reducing the object set of a formal decision context without losing the decision rule information provided by the entire set of objects. Using concept lattices, we propose a theoretical framework of object compression for formal decision contexts. And under this framework, it is proved that the set of all the non-redundant decision rules obtained from the reduced database is sound and complete with respect to the initial formal decision context. Furthermore, a complete algorithm is developed to compute a reduct of a formal decision context. The analysis of some real-life databases demonstrates that the proposed object compression method can largely reduce the size of a formal decision context and it can remove much more objects than both the techniques of clarified context and row reduced context.

1. Introduction

The discovery of previously unknown and potentially useful knowledge from databases is of great importance in the processing and utilization of voluminous information. *Formal concept analysis* (FCA), proposed by Wille [55] in 1982, is one of the effective mathematical tools for knowledge discovery [48–50,56]. FCA starts with a *formal context* defined as a triple (G, M, I) consisting of a set G of objects, a set M of attributes, and a binary relation $I \subseteq G \times M$ indicating that each object of G has what attributes in M. A formal context in FCA corresponds to a special *information system* with two-valued input data in *rough set theory* [39]. FCA organizes the knowledge discovered from a formal context through *concept lattice* whose elements are *formal concepts*, while rough set theory discovers the knowledge from an information system via lower and upper approximations, positive, boundary and negative regions [63]. In fact, there are strong connections between FCA and rough set theory, and some studies have been devoted to comparing and combining these two useful theories [20,21,59].

The construction of concept lattices is an important issue in FCA and much attention has been paid to this issue. For example, many kinds of approaches have been proposed for building different types of concept lattices such as the Wille's concept lattices [15,26,37,38], the *fuzzy concept lattices* [3–5,34,42,51], the *variable threshold concept lattices* [64], the *real concept lattices* [19,31,58] and the *rough concept lattices* [7,60]. Nowadays, the concept lattice theory has been applied to a variety of areas such as machine learning and software engineering [6,23,44,46].

Considering that the concept lattice derived directly from an original formal context is often very complicated, many researchers [1,10,13,14,18,33,35,36,57,66] have studied the issue of reducing the size of a formal context by removing redundant objects or attributes to improve the efficiency of constructing the concept lattice and the understandability of the resulting concept lattice. In FCA, removing the redundant objects of a formal context is called object compression [14] and removing the redundant attributes of a formal context is called attribute compression [14] or feature selection [13]. Since the conciseness of the *attribute implications* [14,16,17,49] and the *association rules* [2,47,62] obtained from a formal context is affected by the number of attributes and the efficiency of extracting these rules from a formal context is additionally affected by the number of objects, both object compression and attribute compression are of great importance in acquiring the rules from a formal context [14].

Moreover, the formal context (G, M, I) with a target attribute $d \notin M$, called *training context*, was introduced by Kuznetsov

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[22,24,25] into FCA for learning from positive and negative examples. Similarly, the formal context (G, M, I) with some target attributes $d_1, \ldots, d_k \notin M$, called *formal decision context*, was proposed by Zhang and Qiu [65] to make decision analysis. A formal decision context (or a training context) is often denoted by a quintuple (G, M, I, N, J) with $N := \{d_1, \ldots, d_k\}$ and $J \subseteq G \times N$. Note that its conditional concept lattice (extracted from (G, M, I)) and decision concept lattice (extracted from (G, N, J)) can jointly be used to express knowledge in the form of the *decision rules* [28–30,57]. Like the case of formal contexts, attribute compression in formal decision contexts has attracted much attention in recent years. The existing attribute compression approaches for formal decision contexts can broadly be divided, according to different compression objectives, into two classes: one is to remove such attributes that are not essential to the rule acquisition [28,30,57]; the other is to remove as many attributes as possible subject to preserving the predefined consistencies [29,41,45,52,54]. Nevertheless, from the viewpoint of rule acquisition, there may also be some non-essential objects in a formal decision context. Especially in the formal decision context with a large number of objects, the non-essential objects in terms of rule acquisition may be far more than the essential ones. Thus, object compression is worth being investigated in formal decision contexts since it allows us, on one hand, to largely cut the cost of storage of the databases and, on the other hand, to make the rule acquisition easier. However, to the best of our knowledge, little work has been done on object compression in formal decision contexts.

In this paper, using concept lattices, we develop an object compression framework for formal decision contexts and formulate a corresponding algorithm to search for a reduct. The proposed object compression method can guarantee that the set of all the non-redundant decision rules obtained from the reduced database is sound and complete with respect to the initial formal decision context, and therefore it can reduce the size of the initial formal decision context without information loss on the decision rules. Furthermore, we prove that the proposed object compression algorithm is complete.

The rest of this paper is organized as follows: In Section 2, we briefly review some basic notions related to FCA. In Section 3, we introduce the notions of object subcontexts and their concept lattices, and give some useful properties. In Section 4, a rule-preserved object compression framework is proposed for formal decision contexts by using concept lattices. In Section 5, we first discuss some useful characteristics of objects in the proposed object compression framework, and then derive a complete algorithm to find a reduct of a formal decision context. In Section 6, some practical databases are analyzed to show the application and scalability of the proposed object compression method. The paper is then concluded with a brief summary.

2. Preliminaries

Definition 1 [55]. A formal context is a triple (G, M, I), where $G := \{x_1, x_2, \ldots, x_m\}$ is a set of objects, $M := \{a_1, a_2, \ldots, a_n\}$ is a set of attributes, and I is a binary relation on $G \times M$ with $(x, a) \in I$ indicating that the object x has the attribute a and $(x, a) \notin I$ indicating the opposite.

A formal context can easily be represented by a two-dimensional table with its input data being numbers 1 and 0, where number 1 on the cross of row and column means that the object in the row has the attribute in the column, and number 0 means the opposite.

In this paper, the concerned formal contexts (G, M, I) are all assumed to be *regular* [66]. That is, for any $(x, a) \in G \times M$, (1) there exist $a_1, a_2 \in M$ such that $(x, a_1) \in I$ and $(x, a_2) \notin I$; (2) there exist $x_1, x_2 \in G$ such that $(x_1, a) \in I$ and $(x_2, a) \notin I$. Such way of regularization causes no effect on the analysis of the formal context (G, M, I) [27].

Definition 2 [55]. Let (G, M, I) be a formal context. For $X \subseteq G$ and $B \subseteq M$, two derivation operators are defined as:

$$\begin{aligned} X^{\uparrow} &:= \{a \in M \mid \forall x \in X, (x, a) \in I\}, \\ B^{\downarrow} &:= \{x \in G \mid \forall a \in B, (x, a) \in I\}. \end{aligned} \tag{1}$$

A pair (X, B) is called a formal concept of (G, M, I) if $X^{\uparrow} = B$ and $B^{\downarrow} = X$. In this case, X and B are called the extent and intent of (X, B), respectively.

When the formal concepts of a formal context (G, M, I) are ordered by:

$$(X_1, B_1) \leqslant (X_2, B_2) :\iff X_1 \subseteq X_2 \ (\iff B_2 \subseteq B_1), \tag{2}$$

they form a complete lattice which is called the concept lattice of (G, M, I) and is denoted by $\mathfrak{B}(G, M, I)$ [14,55]. In the concept lattice $\mathfrak{B}(G, M, I)$, the *infimum* $(\wedge)$ and the *supremum* $(\vee)$ of two formal concepts (X_1, B_1) and (X_2, B_2) are defined by:

$$\begin{aligned} (X_1, B_1) \wedge (X_2, B_2) &= (X_1 \cap X_2, (B_1 \cup B_2)^{\downarrow\uparrow}), \\ (X_1, B_1) \vee (X_2, B_2) &= ((X_1 \cup X_2)^{\uparrow\downarrow}, B_1 \cap B_2). \end{aligned} \tag{3}$$

It can be verified that the pair $(\uparrow, \downarrow)$ of the mappings $\uparrow: 2^G \to 2^M$ and $\downarrow: 2^M \to 2^G$ forms a *Galois connection* [14] between the posets $(2^G, \subseteq)$ and $(2^M, \subseteq)$, where 2^G and 2^M denote the power sets of G and M, respectively. Moreover, the compositions $\uparrow\downarrow: 2^G \to 2^G$ and $\downarrow\uparrow: 2^M \to 2^M$ are *closure operators* [14] on G and M, respectively.

Definition 3 [18]. Let (G, M, I) be a formal context and $X \subset G$. An object $x \notin X$ is said to *be close to* X with respect to (G, M, I) if $\{x\} \cup X$ is an extent of a formal concept of (G, M, I) and the closure $X^{\uparrow\downarrow}$ equals $\{x\} \cup X$.

Definition 4 [16]. Let (G, M, I) be a formal context. An attribute implication of (G, M, I) is an expression $B \to C$, where B and C are subsets of M which are called the premise and conclusion of $B \to C$, respectively. If each $x \in G$ having all the attributes in B also has all the attributes in C, then $B \to C$ is said to be valid in (G, M, I).

For attribute implications $B_1 \to C_1$ and $B_2 \to C_2$, if $B_1 \subseteq B_2$ and $C_2 \subseteq C_1$, we say that $B_1 \to C_1$ implies $B_2 \to C_2$. This inference rule can be characterized by:

$$\frac{B_1 \to C_1, B_1 \subseteq B_2, C_2 \subseteq C_1}{B_2 \to C_2},$$

which is defined to study redundancy of attribute implications with respect to an attribute implication set Ω. More specifically, a non-redundant attribute implication of Ω means that it cannot be implied by others from Ω.

Definition 5 ([14,43]). Let (G, M, I) be a formal context. A set Ω of non-redundant attribute implications is said to be sound and complete with respect to (G, M, I) if each $B \to C \in \Omega$ is valid in (G, M, I) and any valid attribute implication of (G, M, I) can be implied from Ω by means of the above inference rule.

A set Ω of attribute implications is called an α-*inference* base [43] of a formal context (G, M, I) if Ω is sound and complete with respect to (G, M, I). Note that an α-*inference* base is of the same