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A new approach to the rule-base evidential reasoning in the intuitionistic fuzzy setting

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ABSTRACT

A new approach to the rule-base evidential reasoning based on the synthesis of fuzzy logic, Atannasov's intuitionistic fuzzy sets theory and the Dempster-Shafer theory of evidence is proposed. It is shown that the use of intuitionistic fuzzy values and the classical operations on them directly may provide counter-intuitive results. Therefore, an interpretation of intuitionistic fuzzy values in the framework of Dempster-Shafer theory is proposed and used in the evidential reasoning. The merits of the proposed approach are illustrated with the use of developed expert systems for diagnostics of type 2 diabetes. Using the real-world examples, it is shown that such an approach provides reasonable and intuitively obvious results when the classical method of rule-base evidential reasoning cannot produce any reasonable results.

1. Introduction

The methods of rule-base evidential reasoning are based on the synthesis of the tools of Fuzzy Sets theory (*FST*) and the Dempster-Shafer theory (*DST*). The integration of *FST* and *DST* within symbolic, rule-based models primarily was used for solving control and classification problems [5,6,23,36,41]. These models combine these theories in a synergic way, preserving their strengths while avoiding disadvantages they present when used as monostrategy approaches. Generally, such a rule-base evidential reasoning system may be presented as in [6]:

IF $((A \text{ is } L)$ *and* $(B \text{ is } M))$ *THEN* C *is* m_0,
IF $((A \text{ is } H)$ *and* $(B \text{ is } L))$ *THEN* C *is* m_1,

where m_0 and m_1 are two credibility structures with two focal elements and variable C is defined in the universe of discourse which usually is a set of classes to deal with in considered classification problem.

In the above example adopted from [6], the credibility structures were presented as follows:

$$m_0 : D_{00} = \left\{\frac{\mu_{00}^0}{y_0}, \frac{\mu_{01}^0}{y_1}\right\},\ m_0(D_{00}),\ D_{01} = \left\{\frac{\mu_{01}^1}{y_1}\right\},\ m_0(D_{01});$$

$$m_1 : D_{10} = \left\{\frac{\mu_{11}^0}{y_1}, \frac{\mu_{12}^0}{y_2}\right\},\ m_1(D_{10}),\ D_{11} = \left\{\frac{\mu_{10}^1}{y_0}, \frac{\mu_{11}^1}{y_1}, \frac{\mu_{12}^1}{y_2}\right\},\ m_1(D_{11}),$$

where $D_{00}, D_{01}, D_{10}, D_{11}$ are fuzzy subsets in $Y = (y_0, y_1, y_2)$; $\mu_{00}^0,\ \mu_{01}^0,\ \mu_{01}^1,\ \mu_{11}^0,\ \mu_{12}^0,\ \mu_{10}^1,\ \mu_{11}^1,\ \mu_{12}^1$ are the corresponding membership grades, $m_0(D_{00}), m_0(D_{01}), m_1(D_{10}), m_1(D_{11})$ are the basic probability values associated with fuzzy subsets $D_{00}, D_{01}, D_{10}, D_{11}$. The output of the system is obtained in [6] with use of COA method [37]) in the form of defuzzified value $\bar{y}$.

These approaches seem to be justified in the solution of control and classification problems when outputs can be presented by real values.

On the other hand, if we deal with decision support systems, system's outputs can be only the names or labels of corresponding actions or decisions, e.g., the names of medical diagnoses. It is clear that in such cases, the methods based on conventional fuzzy logic, developed for the controlling cannot be used at least directly. A more suitable for the building decision support systems seems to be the so-called *RIMER* approach proposed in [39,40] based on the Evidential Reasoning approach [38].

In the belief rule system, each possible consequent of a rule is associated with a belief degree. Such a rule base is capable to capture more complicated and continuous causal relationships between different factors than traditional *IF-THEN* rules. Therefore, the traditional *IF-THEN* rules may be treated as a special cases of the more general belief rule systems [20,25,30]. In the framework of rule-base inference methodology, using the evidential reasoning (*RIMER*) approach [39] a belief *IF-THEN* rule, e.g., the kth rule R_k, is expressed as follows:

IF $(X_1 \text{ is } A_1^k)\ \Lambda\ (X_2 \text{ is } A_2^k)\ \Lambda \ldots \Lambda\ (X_{T_k} \text{ is } A_{T_k}^k)$
THEN $(D_1, \beta_{1k}), (D_2, \beta_{2k}), \ldots, (D_N, \beta_{Nk})$, with rule weights θ_k, $k = 1$ to L, and attribute weights $\delta_1, \delta_2, \ldots, \delta_{T_k}$, where $A_i^k, i = 1$

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to T_k is the referential value of the ith antecedent attribute, T_k is the number of antecedent attributes used in the kth rule, β_{ik}, $i = 1$ to N, is the belief degree to which D_i is believed to be the consequent of kth antecedent, L is the number of rules in the rule-base, Λ denotes t-norm. If $\sum_{i=1}^{N}\beta_{ik} = 1$, the kth rule is said to be complete; otherwise, it is incomplete. The case of $\sum_{i=1}^{N}\beta_{ik} = 0$ corresponds to the total ignorance about the output given the input in the kth rule. This rule is also referred to as a belief rule.

In the framework of *RIMER* approach, the final outcome obtained as the aggregation of belief rules is presented as $O = \{(D_j, \beta_j)\}$, where β_j, $j = 1$ to N, is the aggregated degree of belief in the decision (hypothesis, action, diagnosis) D_j.

Therefore, the decision characterised by the maximal aggregated degree of belief is the best choice. So the *RIMER* approach can be used for building decision support systems. Nevertheless, there are two restrictions in the *RIMER* approach that reduce its ability to deal with uncertainties that decision makers often meet in practice.

The first restriction is that in the framework of *RIMER* approach, a degree of belief can be assigned only to a particular hypothesis, not to a group of them, whereas the assignment of a belief mass to a group of events is a key principle of the *DST*.

The second restriction is concerned with the observation that in many real-world decision problems we deal with different sources of evidence and the combination of them is needed. The *RIMER* approach does not provide a technique for the combination of evidence from different sources.

It is important that usually the advantages of the approaches based on the rule-base evidential reasoning were demonstrated using simple numerical examples and only relatively small number of examples of solving real-world problems using these approaches were found in the literature [6,22,29,35]. In [42], a novel updating algorithm for *RIMER* model is proposed based on iterative learning strategy for delayed coking unit (*DCU*). Daily *DCU* operations under different conditions are modelled by a belief rule-base, which is then updated using iterative learning methodology, based on a novel statistical utility for every belief rule. The paper [33] presents a hybrid evidential reasoning (*ER*) and belief rule-based (*BRB*) methodology for consumer preference prediction and a novel application to orange juices. In [19], a novel combination of fuzzy inference system and Dempster-Shafer Theory is applied to brain Magnetic Resonance Imaging for the purpose of segmentation where the pixel intensity and the spatial information are used as features. The authors of paper [32] proposed a specific algorithm-Evidential Reasoning based Classification algorithm to recognise human faces under class noise conditions. The methods used in these papers are charged with two above mentioned restrictions of *RIMER* approach. The method used in [24] is free of the second restriction while the first one is retained.

In [16,18,27], a new approach free of both above mentioned restrictions was developed and used for the solution of real-world problems.

It is important that in all above mentioned approaches to the rule-base evidential reasoning, the conventional fuzzy logic was used. For example, the following rule may be used: *If x is Low Then D*, where *Low* is some fuzzy class defined by the corresponding membership function $\mu_{Low}(x)$, D is a name of decision. Nevertheless, in practice we often deal with the intersecting fuzzy classes, e.g., *Low* and *Middle*, and therefore we often have $\mu_{Low}(x) > 0$ and $\mu_{Middle}(x) > 0$. Then if $\mu_{Low}(x) > \mu_{Middle}(x)$ we state that *x is Low* and information of non-zero $\mu_{Middle}(x)$ is lost, whereas the difference between $\mu_{Middle}(x)$ and $\mu_{Low}(x)$ may be very small.

In the current paper, we will show that such a loss of information may lead to incorrect results in the rule-base evidential reasoning and a new method for the solution of these problems based in the synthesis of Atanassov's intuitionistic fuzzy sets (*A-IFS*) [1] and *DST* will be proposed.

In our recent paper [15], we have shown that there exists also a strong link between *A-IFS* and *DST*. This link makes it possible to use directly the Dempster's rule of combination to aggregate local criteria presented by *IFVs* in multiple criteria decision making problems (*MCDM*). The usefulness of the developed method was illustrated using the known example of *MCDM* problem. In [17], we have shown that the classical arithmetical operations on intuitionistic fuzzy values *IFVs* have some limitations (drawbacks) which can lead to incorrect results in applications of *A-IFS* in different fields. Therefore, in [17] using interpretation of *A-IFS* in the framework of *DST*, a set of new operations on *IFVs* treated as belief intervals was proposed and it was proved that these operations are free of limitations (drawbacks) of the classical operations on *IFVs*.

To make the presentation of our approach more transparent, we shall use as an illustration the simple enough, but real-world problem of diagnostic of type-2 diabetes.

For these reasons, the rest of paper is set out as follows. Section 2 presents the basic definition of *A-IFS* and *DST*, the commonly used arithmetical operations on *IFVs* (with their limitations and drawbacks) and introduced in [17] new operations on *IFVs* in the framework of *DST* needed for the subsequent analysis. In Section 3, we present our new approach to the rule-base evidential reasoning based on the synthesis of *A-IFS* and *DST* and perform its advantages using examples obtained with the use of expert system for diagnostics of type 2 diabetes developed on the base of our approach. Finally, the concluding section summarises the paper.

2. Preliminaries

2.1. The basics of A-IFS and problems concerned with operations on IFVs

The concept of *A-IFS* (the reasons for such a notation are presented in [14]) is based on the simultaneous consideration of membership μ and non-membership ν of an element of a set to the set itself (see formal definition in [1]). It is postulated that $0 \leqslant \mu + \nu \leqslant 1$. Following to [1], we call $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ the hesitation degree of the element x in the set A. Hereinafter, we shall call an object $A = \langle \mu_A(x),\ \nu_A(x)\rangle$ intuitionistic fuzzy value (*IFV*).

The operations of addition $\oplus$ and multiplication $\otimes$ on *IFVs* were defined by Atanassov [2] as follows. Let $A = \langle \mu_A, \nu_A\rangle$ and $B = \langle \mu_B, \nu_B\rangle$ be *IFVs*. Then

$$A \oplus B = \langle \mu_A + \mu_B - \mu_A\mu_B, \nu_A\nu_B\rangle, \tag{1}$$

$$A \otimes B = \langle \mu_A\mu_B, \nu_A + \nu_B - \nu_A\nu_B\rangle. \tag{2}$$

These operations were constructed in such a way that they produce *IFVs*. Using operations (1) and (2), in [12] the following expressions were obtained for any integer $n = 1, 2, \ldots$:

$$nA = A \oplus \ldots \oplus A = \langle 1 - (1 - \mu_A)^n, \nu_A^n\rangle,\ A^n = A \otimes \ldots \otimes A = \langle \mu_A^n, 1 - (1 - \nu_A)^n\rangle.$$

It was proved later that these operations produce *IFVs* not only for integer n, but also for all real values $\lambda > 0$, i.e.

$$\lambda A = \langle 1 - (1 - \mu_A)^\lambda, \nu_A^\lambda\rangle, \tag{3}$$

$$A^\lambda = \langle \mu_A^\lambda, 1 - (1 - \nu_A)^\lambda\rangle. \tag{4}$$

The operations (1)–(4) have good algebraic properties [34]:

Let $A = \langle \mu_A, \nu_A\rangle$ and $B = \langle \mu_B, \nu_B\rangle$ be *IFVs*. Then