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Survey on rank preservation and rank reversal in data envelopment analysis

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ABSTRACT

In most data envelopment analysis (DEA) models, the best performers have the full efficient status denoted by unity (or 100), and, from experience, we know that usually plural decision making units (DMUs) have this efficient status. Discriminating between these efficient DMUs is an interesting subject, and a large number of methods have been proposed for fully ranking both efficient and inefficient DMUs. This paper demonstrates the fact that the rank reversal phenomenon may occur in most DEA ranking methods; however, this study introduces some ranking methods which do not follow the procedure and lack this taint. Numerical examples are provided to clearly illustrate the above mentioned phenomenon in some DEA ranking methods. In fact, certain ranking methods are surveyed in DEA focusing on rank preservation and rank reversal phenomena.

1. Introduction

Data envelopment analysis (DEA) was initiated in 1978 when Charnes, Cooper, and Rhodes demonstrated how to change a fractional linear measure of efficiency into a linear programming (LP) format [12]. DEA as a linear programming method can simultaneously take into account multiple inputs and outputs to measure the relative efficiency for the homogenous DMUs in various contexts. In particular, the DEA model is a non-parametric model that does not require the assignment of predetermined weights to input and output factors. DEA has been applied for efficiency measurement in various public and private sectors, including the power industry [15,35,53], education [42], R and D performance [14], health care [4], banking [43,46], the military [12], and courts [30]. Extensive reviews and additional applications are available in Seiford [44] and Charnes et al. [10]. The DEA approach was introduced by Charnes et al. [11];this first model is thus called the Charnes-Cooper-Rhodes (CCR) model. A DEA model is developed to produce an efficiency frontier based on the concept of the Pareto optimum. The DMUs that lie on the efficiency frontier are non-dominated and are, thus, called Pareto-optimal units or efficient DMUs. DMUs that do not lie on the efficiency frontier are deemed to be relatively inefficient. The CCR model assumes constant returns to scale (CRS), implying that the producers are able to linearly scale the inputs and outputs without increasing or decreasing efficiency. Under this assumption, the overall efficiency scores calculated by input-oriented and output-oriented CCR models are equal. Subsequently, Banker et al. [3] proposed the BCC model, which assumes variable returns to scale (VRS). This approach forms a more restricted feasible region than that of the CCR model and, thus, provides Technical Efficiency (TE) scores greater than or equal to those obtained assuming CRS. DEA provides a relative efficiency measure for peer decision making units (DMUs) with multiple inputs and outputs. While DEA has been proven an effective approach in identifying the best practice frontiers, its flexibility in weighting multiple inputs and outputs and its nature of self-evaluation have been criticized. In most models of DEA, the best performers have an efficiency score of unity, and, from experience, we know that there are usually plural DMUs which have this efficient status. Discriminating between these efficient DMUs is an interesting research subject. Several authors have proposed methods for ranking the best performers. These ranking methods have been divided into seven, somewhat overlapping, areas. The first idea, generally known as the super-efficiency method (see, e.g., [1,58,52,38,24]), ranks through the exclusion of the unit being scored from the dual linear program and an analysis of the change in the Pareto frontier. The idea used in the second group is based on benchmarking (see, e.g., [59]), in which a unit is highly ranked if it is chosen as a useful target for many other units. The third group utilizes multivariate statistical techniques, which are generally applied after the DEA dichotomy classification (see, e.g., [20,50,49]).

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The fourth research area ranks inefficient units through proportional measures of inefficiency (see, e.g., [6]). The fifth approach requires the collection of additional, preferential information from relevant decision makers (DMs), and combines multiple criteria decision methodologies with the DEA approach (see, e.g., [22,31]). The sixth research area is based on the concept of common weights analysis (see, e.g., [34,28]). The last area involves the evaluation of a cross-efficiency matrix, in which the units are self- and peer-evaluated (see, e.g., [47,16]).

A question that arises when ranking a group of DMUs is whether there is a change in the ranks of other DMUs after the addition or removal of one or more DMUs. Suppose, for instance, that in the evaluation of the branches of a commercial bank, branch **A** has secured the first rank. Then, another branch with the same inputs and outputs as those of branch **A** is added to the group. Considering that the other branches have not changed, the management of branch **A** would naturally expect to preserve their first place, or at least stand second to the new branch. Contrary to this expectation, however, the new ranking reveals that neither branch **A** nor the new branch has secured the first place. This phenomenon, first observed in MADM (Multiple Attribute Decision Making) ranking models, is called rank reversal. The Analytic Hierarchy Process (AHP), as a very popular multiple criteria decision making (MCDM) approach, has been considerably criticized for its possible rank reversal phenomenon, which means that the relative rankings of two decision alternatives could be reversed when a decision alternative is added or deleted. Such a phenomenon was first noticed and pointed out by Belton and Gear [8], which aroused a long-lasting debate on the validity of AHP and the legitimacy of rank reversal [1–4,10–13,15,16,20–34,36–40].Wang and Luo [60] have shown that the rank reversal phenomenon occurs not only in AHP, but also in many other decision making approaches, such as the BK method for aggregating multiple ordinal preferences, the SAW and the TOPSIS methods for MADM, and the DEA cross-efficiency evaluation method, when a candidate or alternative is added or removed. In this paper, we will demonstrate, by numerical examples that rank reversal occurs in most DEA ranking models. Moreover, some models in which this phenomenon does not occur will also be discussed. In fact, we survey some ranking methods in data envelopment analysis with focus on rank reversal and rank preservation phenomenon. The remainder of the paper is organized into four Sections. In Section 2, we briefly introduce the background of DEA. In Section 3, we demonstrate the rank reversal phenomenon in the cross-efficiency method [47], the super-efficiency models [1,58,38,26,29], and the common weights method [34]. A discussion of some models in which rank reversal does not Section 4. And, finally, the paper is concluded in Section 5.

2. Rank reversal in some DEA ranking methods

In this section, we show through numerical examples that the rank reversal phenomenon also occurs in many DEA ranking approaches, such as the cross-efficiency method [47], super-efficiency models [1,58,38,26,29], and the common weights method [34].

2.1. Rank reversal in DEA cross-efficiency evaluation

The cross-evaluation method was first developed by Sexton et al. [47]. It was developed as a DEA extension tool that can be utilized to identify best performing DMUs and to rank DMUs using cross-efficiency scores that are linked to all DMUs. The main idea of cross-evaluation is to use DEA in a peer-evaluation rather than a self-evaluation mode. Indeed, as Doyle and Green [17] argued, decision makers do not always have a reasonable mechanism by which to choose assurance regions; thus, they recommended the cross-evaluation matrix for ranking units.

Suppose we have a set of n DMUs, and each DMUj produces s different outputs from m different inputs. The ith input and rth output of DMUj (j = 1, 2, …, n) are denoted by $x_{ij}(i = 1,\dots,m)$ and y_{rj} $(r = 1,\dots,s)$, respectively. Cross-efficiency is often calculated in a twophase process. The first phase is carried out using a standard DEA model, e.g., the CCR model. Specifically, for any DMUo under evaluation, E^*_{oo}, the efficiency score under the CCR model, is given by the following optimization problem:

$$
\begin{aligned}
E^*_{oo} = \quad & \max E_{oo} = \frac{\sum_{r=1}^{s} u_{ro}y_{ro}}{\sum_{i=1}^{m} v_{io}x_{io}} \\
& S.t \\
& E_{oo} = \frac{\sum_{r=1}^{s} u_{ro}y_{ro}}{\sum_{i=1}^{m} v_{io}x_{io}} \leqslant 1 \qquad j = 1,\dots,n \\
& u_{ro} \geqslant 0 \qquad\qquad\qquad r = 1,\dots,s \\
& v_{io} \geqslant 0 \qquad\qquad\qquad i = 1,\dots,m
\end{aligned} \tag{2.1}
$$

where v_{io} and u_{ro} represent the ith input and rth output weights for DMUo. The cross-efficiency of DMUj, using the weights that DMUo has chosen in Model 2.1, is then

$$E_{oj} = \frac{\sum_{r=1}^{s} u^*_{ro}y_{rj}}{\sum_{i=1}^{m} v^*_{io}x_{ij}} \qquad j = 1,\dots,n$$

where $(*)$ denotes optimal values in Model 2.1. For DMUj (j = 1, 2, …, n), the average of all E_{oj} (o = 1, 2, …, n), that is $\bar{E}_J = 1/n\sum_{o=1}^{n} E_{oj}$ is referred to as the cross-efficiency score for DMUj. We point out that the DEA Model 2.1 is equivalent to the following linear program:

$$
\begin{aligned}
E^*_{oo} = \quad & \max E_{oo} = \sum_{r=1}^{s} u_{ro}y_{ro} \\
& S.t \\
& \sum_{i=1}^{m} v_{io}x_{io} = 1 \\
& \sum_{r=1}^{s} u_{ro}y_{ro} - \sum_{i=1}^{m} v_{io}x_{io} \leqslant 0 \quad j = 1,\dots,n \\
& u_{ro} \geqslant 0 \qquad\qquad\qquad r = 1,\dots,s \\
& v_{io} \geqslant 0 \qquad\qquad\qquad i = 1,\dots,m
\end{aligned} \tag{2.2}
$$

Cross-efficiency evaluation has been used in various applications, e.g., efficiency evaluation of nursing homes [47], selection of a flexible manufacturing system [48], technology selection [2], determining the most efficient number of operators and the efficient measurement of labor assignment in a cellular manufacturing system (CMS) [19], measuring the performance of the nations in the Summer Olympic Games [65], evaluating computer numerical control (CNC) machines in terms of system specification and cost [54], system R and D project selection [41,68], extension of the analysis of an efficiency and productivity study on a cellular manufacturing system (CMS) [56], preference voting [23,69], rating decision alternatives [63], and so on. Some studies on other DEA issues are very relevant to the cross-efficiency concept (see, e.g., [39,7,37]). Chen [13] compared technical efficiency and cross-efficiency scores in the electricity distribution sector in Taiwan through a DEA framework. Wu et al. [66,67] used the cross-efficiency evaluation method to measure the performance of the nations participating in the last six Summer Olympic Games. They used the cross-efficiency evaluation method because the cross-efficiency score of a DMU is obtained by computing the DMU's set of n scores (using the n sets of optimal weights), and then averaging those scores [47]. Therefore, cross-efficiency is a better choice for measuring the performance of nations in the Olympic Games. Also, in Wu et al. [64], Liang et al.'s [33] DEA game cross-efficiency model was modified and used to measure the performance of the nations participating in the last six Summer Olympic Games. They extended Liang et al.'s [33]