Knowledge-Based Systems 35 (2012) 201–210

Contents lists available at SciVerse ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

# Group consensus based on evidential reasoning approach using interval-valued belief structures

Chao Fu *, Shanlin Yang

*School of Management, Hefei University of Technology, Box 270, Hefei 230009, Anhui, PR China*
*Key Laboratory of Process Optimization and Intelligent Decision-Making, Ministry of Education, Hefei 230009, Anhui, PR China*

ARTICLE INFO

*Article history:*
Received 9 September 2011
Received in revised form 3 May 2012
Accepted 4 May 2012
Available online 14 May 2012

*Keywords:*
Multiple attribute group decision analysis
Evidential reasoning approach
Group consensus
Strategy for renewing assessments
Nonlinear optimization

ABSTRACT

This paper proposes a method to reach required group consensus (GC) and find GC-based solutions to multiple attribute group decision analysis (MAGDA) problems using interval-valued belief structures (IBSs) based on evidential reasoning approach. The GC at the attribute, alternative and global levels is constructed based on IBSs. Subjective weights of experts, weights of attributes, and utilities of experts for assessment grades are extended to intervals. Hereinto, the former two can be characterized by four kinds of relevant constraints, and combined with the constraints to be incorporated into the optimization problems for the GC. Also, utilities of experts for assessment grades with the consistent combination of relevant constraints and their intrinsic constraint contribute to the GC. Further, a strategy for experts to renew assessments is designed to improve the GC. A preferentially developed industry selection problem is solved by the proposed method to demonstrate its detailed implementation process, and its validity and applicability.



## 1. Introduction

In group decision analysis (GDA), a consensus process is very useful to find potential problems and correspondingly improve group decision quality and group satisfaction [11]. To implement the consensus process, many GDA approaches have been proposed in a fuzzy context using linguistic or numerical preferences (e.g. [3,4,7,14–16,19,23,30]). However, these approaches lack consideration of three important factors, including experts' utilities, subjective weights of experts, and the flexibility in group consensus (GC), and further some approaches do not focus on multiple attribute group decision analysis (MAGDA) problems.

Under the framework of Dempster–Shafer theory, like many approaches (e.g. [9,29,31,33]), an evidential reasoning (ER) approach [34–36] was developed to solve uncertain multiple attribute decision analysis (MADA) problems. The ER approach was extended in GDA context to solve MAGDA problems with GC requirements [11]. The three important factors mentioned above are considered in this extension. Further, an attribute weight based feedback model [12] and a consensus framework [13] were designed to accelerate convergence to GC.

In some situations, an account of incompleteness or lack of information, knowledge and data, which results in partial or total ignorance, experts or decision makers may often feel too restrictive and difficult to give precise (crisp) assessments. To deal with these situations, some methods have been developed to express assessments by interval-valued intuitionistic fuzzy numbers [37], interval-valued fuzzy preference relations [21] and interval probability [22]. However, these methods are not designed to reach the required GC and find GC-based solutions to MAGDA problems with GC requirements.

In this paper, a method is developed based on the ER approach to consider interval-valued assessments, the three important factors mentioned above and the focus on MAGDA problems. In the method, experts give interval-valued assessments denoted by interval-valued belief structures (IBSs) [8,26,27,32], and the required GC should be reached.

In the literature, IBSs and IBS-based MADA approaches have been investigated. In [8,32], basic concepts of the theory of belief functions including belief and plausibility measures, Dempster's rule of combination and uncertainty measures were extended in the situation of IBSs. In [27], a logical optimality approach was developed to implement reasonable combination and normalization of IBSs. Further, an IBS-based MADA approach in the ER context using the combination method in [27] was designed in [26]. In the MADA approach, the nonlinear optimization problems of constructing expected utilities of alternatives and the generation of a ranking order of alternatives using the expected utilities were also introduced. They can be used to generate GC-based solutions to MAGDA problems with GC requirements, when the required GC is reached and group assessments on each attribute are formed.

* Corresponding author at: School of Management, Hefei University of Technology, Box 270, Hefei 230009, Anhui, PR China. Tel.: +86 0551 2904930; fax: +86 0551 2905263.

*E-mail address:* wls_fuchao@163.com (C. Fu).

0950-7051/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.knosys.2012.05.001

However, these researches obviously do not focus on the process of reaching the required GC, although they are related to IBSs.

In the proposed method, GC becomes an interval rather than a precise value in [11]. The lower and upper bounds of GC are required to reach. To check the lower and upper bounds of GC, GC is firstly constructed at three levels, consisting of the attribute, alternative and global levels. The GC at the attribute level is generated by solving a pair of nonlinear optimization problems constructed based on the precise GC at the attribute level in [11]. The GC at the alternative and global levels derives from the GC at the attribute level. Then, interval-valued threshold vector and thresholds are set to check whether the required GC at three levels is reached.

Because subjective weights of experts, weights of attributes, and utilities of experts for assessment grades are considered to construct precise GC in [11], they also influence the lower and upper bounds of GC in the proposed method. Similar to experts' assessments, due to incompleteness or lack of information, knowledge and data, they can be extended as intervals and characterized by four kinds of relevant constraints, i.e. fuzzy preference constraint, multiplicative preference constraint, linear inequality constraint and ordinal inequality constraint. Interval-valued subjective weights of experts, weights of attributes, utilities of experts for assessment grades, and their relevant constraints can be incorporated into the optimization problems for constructing the GC at three levels. When the required GC is reached, they further give a contribution to GC-based solutions. Particularly, they can be applied solely or simultaneously, which enhance the applicability and flexibility of the proposed method in practice.

In the process of reaching the required GC in the proposed method, several rounds of group analysis and discussion (GAD) are organized by manager. After each GAD, experts renew their assessments to simultaneously improve the lower and upper bounds of GC, or to improve one bound and at the same time keep the other bound. Also, renewed assessments should be valid and normalized, and their imprecision should be controlled effectively. This is obviously more complex than renewing precise assessments in [11]. In the proposed method, a strategy is designed and recommended for experts to reasonably renew assessments with the aim of improving GC on the condition that the renewed assessments are valid, normalized and imprecision-controlled.

The rest of this paper is organized as follows. Section 2 presents preliminaries related to the proposed method. Section 3 interprets the proposed method in detail. A preferentially developed industry selection problem is solved in Section 4 to demonstrate a detailed implementation process of the proposed method, its validity and applicability. Section 5 discusses the situation of interval-valued subjective weights of experts, weights of attributes, utilities of experts for assessment grades, and their relevant constraints in the problem in Section 4, and compares the proposed method with other methods. Finally, this paper is concluded in Section 6.

## 2. Preliminaries

### 2.1. The ER distributed modeling framework for MAGDA problems using IBSs

Suppose a MAGDA problem includes $T$ experts $t_j(j = 1,\ldots,T)$ and a manager. The relative weights of $T$ experts on the attribute $e_i(i = 1,\ldots,L)$ for the alternative $a_l(l = 1,\ldots,M)$ are denoted by $\lambda(e_i(a_l)) = (\lambda^1(e_i(a_l)), \lambda^2(e_i(a_l)),\ldots,\lambda^T(e_i(a_l)))$ such that $0 \leqslant \lambda^j(e_i(a_l)) \leqslant 1$ and $\sum_{j=1}^{T}\lambda^j(e_i(a_l)) = 1$.

It is demonstrated in [11] that weights of experts consist of two parts, which are subjective weights and objective weights. Subjective weights of experts reflect the difference among background, experience and knowledge of experts; and objective weights of experts show the difference among experts' assessments in generating group assessment.

The MAGDA problem has $M$ alternatives $a_l(l = 1,\ldots,M)$, on the upper level attribute, referred to as a general attribute, and $L$ lower level attributes $e_i(i = 1,\ldots,L)$, called basic attributes. The relative weights of $L$ basic attributes are denoted by $w = (w_1,w_2,\ldots,w_L)$ such that $0 \leqslant w_i \leqslant 1$ and $\sum_{i=1}^{L} w_i = 1$.

Suppose $H_n(n = 1,\ldots,N)$ denotes a set of grades which forms the frame of discernment $\Omega = \{H_1,H_2,\ldots,H_N\}$. $M$ alternatives are assessed at $L$ attributes using $H_n(n = 1,\ldots,N)$. Let $B^j(e_i(a_l)) = \left\{\left(H_n, \left[\beta^{j-}_{n,i}(a_l), \beta^{j+}_{n,i}(a_l)\right]\right),\ n = 1,\ldots,N\right\}$ denote the interval-valued distributed assessment vector given by the expert $t_j$ on the attribute $e_i$ for the alternative $a_l$ to the grade $H_n$ with the interval belief degree of $\left[\beta^{j-}_{n,i}(a_l), \beta^{j+}_{n,i}(a_l)\right]$. The interval belief degree satisfies $\beta^{j-}_{n,i}(a_l) \geqslant 0, \beta^{j-}_{n,i}(a_l) \leqslant \beta^{j+}_{n,i}(a_l), \sum_{n=1}^{N}\beta^{j-}_{n,i}(a_l) \leqslant 1, \beta^{j}_{n,i}(a_l) \in \left[\beta^{j-}_{n,i}(a_l), \beta^{j+}_{n,i}(a_l)\right]$, $\beta^{j-}_{\Omega,i}(a_l) = max\left(0, 1-\sum_{n=1}^{N}\beta^{j+}_{n,i}(a_l)\right), \beta^{j+}_{\Omega,i}(a_l) = 1-\sum_{n=1}^{N}\beta^{j-}_{n,i}(a_l), \beta^{j}_{\Omega,i}(a_l) \in \left[\beta^{j-}_{\Omega,i}(a_l), \beta^{j+}_{\Omega,i}(a_l)\right]$ and $\sum_{n=1}^{N}\beta^{j}_{n,i}(a_l) + \beta^{j}_{\Omega,i}(a_l) = 1$, where $\beta^{j}_{\Omega,i}(a_l)$ and $\left[\beta^{j-}_{\Omega,i}(a_l), \beta^{j+}_{\Omega,i}(a_l)\right]$ denote the belief degree assigned to $\Omega$ and its interval, respectively. If $\beta^{j}_{\Omega,i}(a_l) = 0$ always holds, then the assessment vector is complete; otherwise, it is incomplete.

## 3. The proposed method

The main study in the proposed method is to reach the required GC, so the GC at three levels based on interval-valued assessments is constructed first. Due to consideration of subjective weights of experts, weights of attributes, and utilities of experts for assessment grades in constructing GC, the situations of interval-valued subjective weights of experts with relevant constraints, interval-valued weights of attributes with relevant constraints, and interval-valued utilities of experts for assessment grades with relevant constraints are handled sequentially. Hybrid situations are also supported by the proposed method, which is demonstrated in Section 5. To help experts renew assessments, a renewing strategy is designed to improve GC and generate valid, normalized and imprecision-controlled assessments. When the required GC is reached after several rounds of GAD, experts' assessments on each attribute are combined to form group assessments on each attribute. Finally, a whole procedure of the proposed method is given.

### 3.1. The GC at three levels

In the proposed method, GC is an interval instead of a precise value due to the fact that experts' assessments are IBSs instead of belief structures (BSs). It is constructed at three levels including the attribute, alternative and global levels. The GC at the attribute level is defined as follows.

**Definition 1.** Suppose the GC on the attribute $e_i(i = 1,\ldots,L)$ for the alternative $a_l(l = 1,\ldots,M)$ is denoted by $[gc^-(e_i(a_l)), gc^+(e_i(a_l))]$ such that $gc(e_i(a_l)) \in [gc^-(e_i(a_l)), gc^+(e_i(a_l))]$. Based on experts' assessments $V^j(e_i(a_l))$ composed of $B^j(e_i(a_l)) = \left\{\left(H_n, \left[\beta^{j-}_{n,i}(a_l), \beta^{j+}_{n,i}(a_l)\right]\right),\ n = 1,\ldots,N\right\}$ and $\left[\beta^{j-}_{\Omega,i}(a_l), \beta^{j+}_{\Omega,i}(a_l)\right]$, the lower and upper bounds of GC can be obtained by solving the following pair of nonlinear optimization problems.