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A new EEG synchronization strength analysis method: S-estimator based normalized weighted-permutation mutual information

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ABSTRACT

Synchronization is an important mechanism for understanding information processing in normal or abnormal brains. In this paper, we propose a new method called normalized weighted-permutation mutual information (NWPMI) for double variable signal synchronization analysis and combine NWPMI with S-estimator measure to generate a new method named S-estimator based normalized weighted-permutation mutual information (SNWPMI) for analyzing multi-channel electroencephalographic (EEG) synchronization strength. The performances including the effects of time delay, embedding dimension, coupling coefficients, signal to noise ratios (SNRs) and data length of the NWPMI are evaluated by using Coupled Henon mapping model. The results show that the NWPMI is superior in describing the synchronization compared with the normalized permutation mutual information (NPMI). Furthermore, the proposed SNWPMI method is applied to analyze scalp EEG data from 26 amnestic mild cognitive impairment (aMCI) subjects and 20 age-matched controls with normal cognitive function, who both suffer from type 2 diabetes mellitus (T2DM). The proposed methods NWPMI and SNWPMI are suggested to be an effective index to estimate the synchronization strength.

1. Introduction

Synchronization has been suggested to be a key indicator to the information process in normal or abnormal brains (e.g. Dubovik, Bouzerda-Wahlen, & Nahum, 2013; Liang, Liang, & Wang, 2015; Liang, Wang, & Ou Yang, 2013; Tóth, File, & Boha, 2014). Recent studies show that a lot of intractable neurological diseases such as epilepsy, Alzheimer's, Parkinson's disease, schizophrenia, autism are associated with the abnormal electroencephalographic (EEG) synchronization (Albada, Gray, & Sdale, 2009; Albada & Robinson, 2009; Cui, Liu, & Bian, 2014; Cui, Liu, & Wan, 2010). EEG synchronization analysis is a promising approach to investigate synchronization in the brain and to study the mechanism of neurological diseases. Therefore, how the synchronization strength index of multivariate neural signals is estimated has become a crucial issue.

A considerable amount of synchronization algorithms have been developed to detect the change of EEG synchronization, including Mutual information (MI) (Aydin, Tunga, & Yetkin, 2015; Liu, Huang, & Chou, 2012; Melia, Guaita, & Vallverdú, 2015), Phase Synchronization (Cantero & Atienza, 2009), Coherence (Dubovik et al., 2013; Moretti, Frisoni, & Pievani, 2008; Nunez, Silberstein, & Shi, 1999; Nunez, Srinivasan, & Westdorp, 1997), Graph Theoretical Analysis (De, Wm, & Koene, 2012; Schependom, Gielen, & Laton, 2014; Yang & Lin, 2015), Granger Causality (Babiloni, Ferri, & Binetti, 2009; Dauwels, Vialatte, & Musha, 2010), the Statistics Event Synchronous measurement (Dauwels et al., 2010), Phase Lag Index (PLI) (Bousleiman, Zimmermann, & Ahmed, 2014; Tóth et al., 2014), Global Synchronization Index (GSI) (Cui et al., 2010; Lee, Park, & Kim, 2010), Global Field Synchronization (GFS) (Ma, Liu, & Liu, 2014; Park, Che, & Im, 2008), Global Coupling Index (GCI) (Wen, Xue, & Lu, 2014) and so on. Among these methods, MI is one of the most significant information theoretic interdependence measures. An improved method, permutation mutual information (PMI) (Ou Yang, 2010) has been proposed and successfully applied to study the EEG synchronization. The PMI can calculate probability distribution density of ordinal patterns simply and effectively in high-dimensional spaces, and then directly estimate the mutual information between the bivariate time series. In addition, it does better in resisting noise interference than traditional histogram method.

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Although PMI has been used widely, there remain some unresolved problems. When the ordinal patterns of each time series are extracted, the order structure is the only information retained and a lot of amplitude information is lost. To resolve this problem, we propose a new method called normalized weighted-permutation mutual information (NWPMI) to quantify synchronization strength of EEG. In this method, a different approach to extract the patterns from a given time series by incorporating amplitude information is suggested. To evaluate the performances of this new method, Coupled Henon mapping model is applied to generate double-channel time series. The effects of different time delay, embedding dimension, coupling coefficients, signal to noise ratios (SNRs) and data length in the method are investigated. Furthermore, the NWPMI method and S-estimator are combined into a new global synchronization measure which is named S-estimator based normalized weighted-permutation mutual information (SNWPMI). Finally, the new global synchronization method is applied to analyze the synchronizations of 26 amnestic mild cognitive impairment (aMCI) subjects and 20 age-matched controls with type 2 diabetes mellitus (T2DM). The correlation between synchronization and cognitive functions is also analyzed.

2. Methods

2.1. Mutual information

The core concept of information theory is the entropy which can be defined as the average amount of code necessary. For a discrete variable X with finite states $x_i \in A$ with probability $p(x_i)$, the Shannon entropy of this set of probabilities is:

$$H(X) = -\sum_{x_i \in A} p(x_i) \log p(x_i). \tag{1}$$

Let us consider a pair of random variables X and Y. MI is the most well-known information theoretic interdependence measure, which is defined as:

$$I(X; Y) = H(X) + H(Y) - H(X, Y) \tag{2}$$

where $H(X)$ and $H(Y)$ is the Shannon entropy of X and Y respectively, and $H(X, Y)$ is the joint entropy of X and Y. The mutual information quantifies the amount of information the random variable Y contains about random variable X (and vice versa).

The normalized mutual information (NMI) is:

$$NMI = I(X; Y)/\min\{H(X), H(Y)\} \tag{3}$$

where $I(X; Y) \leq \min\{H(X), H(Y)\}$. NMI ranges from 0 to 1. When the random variables X and Y are independent, $NMI = 0$; when the random variables X and Y are fully correlated, $NMI = 1$.

2.2. Permutation mutual information

In 2002, Bandt and Pompe proposed a new method for ordinal pattern of time series analysis on "Physical Review Letters" in the United States (Bandt & Pompe, 2002). Based on ordinal pattern analysis method, Ou Yang proposed a new mutual information measure called PMI by comparing neighboring values of each point and mapping them to ordinal patterns (Ou Yang, 2010).

Let us consider time series X and Y. They are embedded into m-dimensional space, in which the time-delay embedding vectors are expressed as $X_i = (x_i, \; x_{i+\tau}, \ldots, x_{i+(m-1)\tau})$ and $Y_i = (y_i, \; y_{i+\tau}, \ldots, y_{i+(m-1)\tau})$, $i = 1, 2, \ldots, L - (m-1)\tau$, where m and τ is the embedding dimension and time delay respectively. The elements of X_i and Y_i are re-sorted separately in ascending order. When the two values of X_i or Y_i are equal, they are sorted based on the size of the subscript. Thus, each of the subvector in m-dimensional space is mapped into an ordinal pattern π_i out of $m!$.

The ordinal patterns of time series X are analyzed firstly. The number of the ordinal patterns $\pi_1, \pi_2, \ldots, \pi_{m!}\pi_i$ is denoted by $C_1, \; C_2, \ldots, C_{m!}$. Then the probability of ordinal pattern π_i can be calculated as

$$p(X = \pi_i) = C_i/(L - (m-1)\tau). \tag{4}$$

The probability distribution p_x of time series X is obtained. Similarly the probability distribution of Y is p_y. For double-channel signals, π_i and π_j are ordinal patterns of X_i and Y_i respectively, thus there are $m!*m!$ kinds of joint ordinal patterns. All the same ordinal patterns are treated as a group, of which the number is denoted by C_{ij}. Then the probability of each joint ordinal pattern can be calculated as

$$p(X = \pi_i, Y = \pi_j) = C_{ij}/(L - (m-1)\tau) \tag{5}$$

p_{xy} is obtained as probability distribution of joint ordinal pattern.

Based on the Shannon information theory, the permutation entropy (PE) and the joint permutation entropy of the time series are defined as:

$$\begin{aligned} PE(X) &= -\sum_{i=1}^{m!} p_x \log(p_x) \\ PE(Y) &= -\sum_{i=1}^{m!} p_y \log(p_y) \\ PE(X, Y) &= -\sum_{i=1}^{m!}\sum_{j=1}^{m!} p_{xy} \log(p_{xy}). \end{aligned} \tag{6}$$

The permutation mutual information (PMI) is defined as

$$PMI(X; Y) = PE(X) + PE(Y) - PE(X, Y). \tag{7}$$

The normalized permutation mutual information (NPMI) is defined as

$$NPMI = PMI(X; Y)/\min\{PE(X), PE(Y)\} \tag{8}$$

where NPMI ranges from 0 to 1. The bigger the value of NPMI is, the stronger the interaction of X and Y is.

2.3. Weighted-permutation mutual information

The main shortcoming of PMI resides in the fact that when the ordinal patterns for each time series are extracted, there is no information retained besides the order structure, therefore a lot of amplitude information is lost. For example, when PMI is calculated, time series 1, 2, 3 and 1, 2, 4 are indicated as the same ordinal pattern. However, these two groups of data are not completely the same, and their contributions to MI should not be exactly the same, either. Fig. 1 shows how the same ordinal pattern can originate from different m-dimensional vectors.

Fadlallah, Chen, and Keil (2013) modified PE procedure by saving the useful amplitude information of the signals, and proposed weighted-permutation entropy (WPE). Here, we proposed Weight Permutation Mutual Information (WPMI) stemming from the similar idea to calculate the MI between two time series.

Consider the time series X and Y. They are embedded into m-dimensional space, each of the subvectors in m-dimensional space is assigned to a ordinal pattern π_i. Ordinal patterns of X are analyzed firstly. The weighted probability p_ω for each ordinal pattern is calculated as:

$$p_\omega(\pi_i) = \frac{\sum I_{\pi_i}(X_i)\omega_i}{\sum I_{\prod}(X_i)\omega_i} \tag{9}$$