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Stability analysis of switched cellular neural networks: A mode-dependent average dwell time approach

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ABSTRACT

This paper addresses the exponential stability of switched cellular neural networks by using the mode-dependent average dwell time (MDADT) approach. This method is quite different from the traditional average dwell time (ADT) method in permitting each subsystem to have its own average dwell time. Detailed investigations have been carried out for two cases. One is that all subsystems are stable and the other is that stable subsystems coexist with unstable subsystems. By employing Lyapunov functionals, linear matrix inequalities (LMIs), Jessen-type inequality, Wirtinger-based inequality, reciprocally convex approach, we derived some novel and less conservative conditions on exponential stability of the networks. Comparing to ADT, the proposed MDADT show that the minimal dwell time of each subsystem is smaller and the switched system stabilizes faster. The obtained results extend and improve some existing ones. Moreover, the validness and effectiveness of these results are demonstrated through numerical simulations.

1. Introduction

Since the seminal work by Chua and Yang (1988a, 1988b), cellular neural networks (CNNs) have been intensively studied (see, for example, Arik and Tavsanoglu (2000), Yuan, Cao, and Deng (2006) and the references therein). This is partially due to the successful hardware implementations for them and their various applications in associative memory, moving object speed detection, pattern classification, fixed point computation, image processing and so on. These applications rely crucially on the stability of the networks (Arik & Tavsanoglu, 2000; Huang & Cao, 2011b; Huang, He, & Wang, 2008; Huang, Huang, & Liu, 2005; Li & Rakkiyappan, 2013; Liu, Lu, & Chen, 2014; Lu, Ho, & Cao, 2011; Rakkiyappan, Chandrasekar, Lakshmanan, & Park, 2014; Rakkiyappan, Zhu, & Chandrasekar, 2014; Yuan et al., 2006). Therefore, a necessary step for practical design of CNNs is the analysis of their dynamic behaviors. In fact, these theoretical results not only improve our understanding of the system's dynamics but also are important complements to experimental and numerical investigations using analog circuits and digital computers.

During hardware implementation, time delays occur due to the finite switching speed of the amplifiers and communication time and hence it is of great importance to incorporate delays in neural networks (Arik & Tavsanoglu, 2000; Cao & Wang, 2005; Hu & Wang, 2015; Huang & Cao, 2011b; Lu et al., 2011; Tang, Gao, Zhang, & Kurths, 2015; Wang, Zhang, & Jiang, 2011; Wang, Zhang, & Yu, 2009a, 2009b; Zhang & Shen, 2014; Zhang, Shen, Yin, & Sun, 2015; Zhang, Tang, Miao, & Du, 2013). A time delayed cellular neural network (DCNNs) is as follows,

$$\dot{u}_i(t) = -a_i u_i(t) + \sum_{j=1}^{n} b_{ij} g_j(u_j(t)) + \sum_{j=1}^{n} c_{ij} g_j(u_j(t-\tau)) + J_i, \quad (1)$$

where $t \geq 0$, $i = 1, 2, \ldots, n$. Here n denotes the number of units in the network; u_i is the voltage or potential of the ith cell; a_i denotes the rate with which the cell resets its potential to the resting state when isolated from other cells and external inputs, b_{ij} and c_{ij} denote the connection and delayed connection weight coefficients of the neurons; g_j is the nonlinear output function; J_i denotes the ith component of an external input source; τ denotes the time delay. Although this model could simulate some simple practical situations when the number of cells is not large, it has

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been well recognized that neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Therefore, it is more reasonable to introduce continuously distributed delays. For neural network models with distributed delays, we refer readers to Huang and Cao (2011b) and Principle, Kuo, and Celebi (1994). Introducing distributed delays into (1) and generalizing the constant delay into time-varying delay, we can obtain the following modified model,

$$\dot{u}(t) = -Au(t) + Bg(u(t)) + Cg(u(t-\tau(t))) + D\int_{t-h(t)}^{t} g(u(s))ds + J, \tag{2}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathbb{R}^n$ is the state vector of the neurons; $A = \text{diag}(a_1, a_2, \ldots, a_n), B = (b_{ij})_{n\times n}, C = (c_{ij})_{n\times n}, D = (d_{ij})_{n\times n}, g(u(t)) = (g_1(u_1(t)), g_2(u_2(t)), \ldots, g_n(u_n(t)))^T.J = (J_1, J_2, \ldots, J_n)^T$, $\tau(t)$ and $h(t)$ are non-negative time-varying functions with $0 \leqslant \tau(t) \leqslant \tau_N, 0 \leqslant h(t) \leqslant h_N$, for some positive constants τ_N and h_N.

Generally, DCNNs are large-scale nonlinear systems consisting of many subsystems. When subsystems switch, existing links between neurons may be cut off and new ones could be established. This would quickly change the connection topology of the system. To obtain a deep and clear understanding of the dynamics of these complex systems, one usually investigate the so called switched DCNNs. Each switched neural networks is composed of a family of continuous-time or discrete-time subsystems and there is a rule orchestrating the switching among the subsystems (Huang, Qu, & Li, 2005). A switched DCNNs associated with (2) is

$$\dot{u}(t) = -A_{\sigma(t)}u(t) + B_{\sigma(t)}g(u(t)) + C_{\sigma(t)}g(u(t-\tau(t))) + D_{\sigma(t)}\int_{t-h(t)}^{t} g(u(s))ds + J, \tag{3}$$

where $\sigma(t) : [0, +\infty) \to \Sigma = \{1, 2, \ldots, N\}$ is the switching signal, which is a piecewise constant and right continuous function of time t, N is the number of subsystems.

The stability problem of switched systems under controlled switching signals has always been a hot topic. In practice, a class of controlled switching signals with restrictions on switching instants is frequently encountered and considerable attention have been drawn to such a type of switching called slow switching (Wu, Shi, Su, & Chu, 2011). One way to specify slow switching is to introduce a scalar and restrict the switching signals with a property that the switching times $t_1, t_2, \ldots$ satisfy $t_{i+1} - t_i \geqslant \bar{\tau}$ for all i belongs to Σ. In other words, all switching signals with intervals between consecutive discontinuities are not smaller than $\bar{\tau}$. The scalar $\bar{\tau}$ is coined as the dwell time. In Hespanha and Morse (1999), the concept of "dwell time" is extended to that of "average dwell time" (ADT). It is shown that a similar result still holds when the intervals of switching signals among consecutive discontinuities are enlarged so that some intervals have lengths less than τ_a but the average interval between consecutive discontinuities is not less than τ_a. In the last few years, considerable efforts have been devoted to the stability analysis of switched systems and various sufficient conditions on stability have been derived by applying the method of ADT (see Lian & Wang, 2015; Lu et al., 2011; Zhang & Gao, 2010 and the references cited therein). A key step in ADT is to construct a suitable Lyapunov functional, which usually is not an easy job.

On one hand, ADT switching means that the number of switches in a finite interval is bounded and the average dwell time between consecutive switching is not less than a common positive constant τ_a. It is obvious that ADT is independent of the state of each subsystem. Since all subsystems are required to have the same ADT and the individual properties of each subsystem are neglected, it is inevitable to bring more conservative results. On the other hand, although all the subsystems are stable, the switched system may be unstable (Lian & Wang, 2015; Lou & Cui, 2008). In practice, switched systems with unstable subsystems are inevitably encountered in real plant, for instance, sensor faults or controller failure can lead to unstable subsystems (Lin & Antsaklis, 2009; Zhao, Zhang, Shi, & Liu, 2012). So far, the majority of existing work only considers switched systems consisting of stable subsystems. Only few papers deal with switched systems composed of both stable and unstable subsystems (Li, Zhao, & Dimirovski, 2009; Long & Zhao, 2015; Zhai, Hu, Yasuda, & Michel, 2001). When some unstable subsystems are included in switched systems, it is hard to design the switching law such that each unstable mode increases within an upper bound during its activating period (Yang, Jiang, & Cocquempot, 2014). Moreover, for practical complex systems, it may be difficult and unnecessary to stabilize all subsystems (Long & Zhao, 2015; Yang et al., 2014). How to stabilize switched nonlinear CNNs with both stable and unstable subsystems is an interesting problem (Tang et al., 2015; Zhang, Tang et al., 2013). The purpose of this study is to establish conditions guaranteeing the exponential stability of such systems. To achieve it, we shall develop the mode-dependent average dwell time method (MDADT). The idea of MDADT has been used to study the stability properties of switched linear systems by some researchers, among whom are Liu, Lian, and Zhuang (2015), Zhang, Xie, Zhang, and Gang (2014) and Zhao et al. (2012).

The main contribution of this paper lies in the following two aspects. In the first place, to the best of our knowledge, this is the first time to develop the method of MDADT to switched nonlinear CNNs systems with time-varying delay. To a large extent, the existing literature on theoretical studies of switched neural networks systems is predominantly concerned with ADT and all subsystems required to be stable. Literature dealing with the switched neural networks systems when stable subsystems coexist with unstable subsystems seems to be scarce. In the second place, by applying the MDADT method, every state can be separated from the whole system among the operation time. Whereas ADT require all subsystems have the same dwell time and the individual properties of each subsystem are neglected, it is inevitable to bring some restriction. The advantages of the proposed MDADT can be characterized by two sides: the minimal dwell time of each subsystem is smaller and the switched system stabilizes faster. Therefore, it is obviously that MDADT is better than ADT and the mechanism of MDADT compensates the shortage of ADT approach greatly when we derive the conditions for stability.

The remainder of this article is organized as follows. In Section 2, we present some preliminaries and the assumptions. In Section 3, sufficient conditions are derived for the exponential stability of the switched DCNNs by using the reciprocally convex approach and techniques of Lyapunov functional, MDADT method, and linear matrix inequalities (LMI). In Section 4, numerical examples are provided to demonstrate the validness and effectiveness of the obtained main results. The paper concludes with some remarks in Section 5.

2. Preliminaries

In this section, we state some notations, definitions and lemmas. In the sequel, $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices, I_n represents the $n \times n$ dimensional identity matrix, and 0_n represents the $n \times n$ dimensional zero matrix. For a square matrix, A^T denotes the transpose of A, $A > 0$ (< 0) means A is positively (negatively) definite, and $\lambda_{min}(A)$ and $\lambda_{max}(A)$ respectively represent the minimum eigenvalue and the maximal